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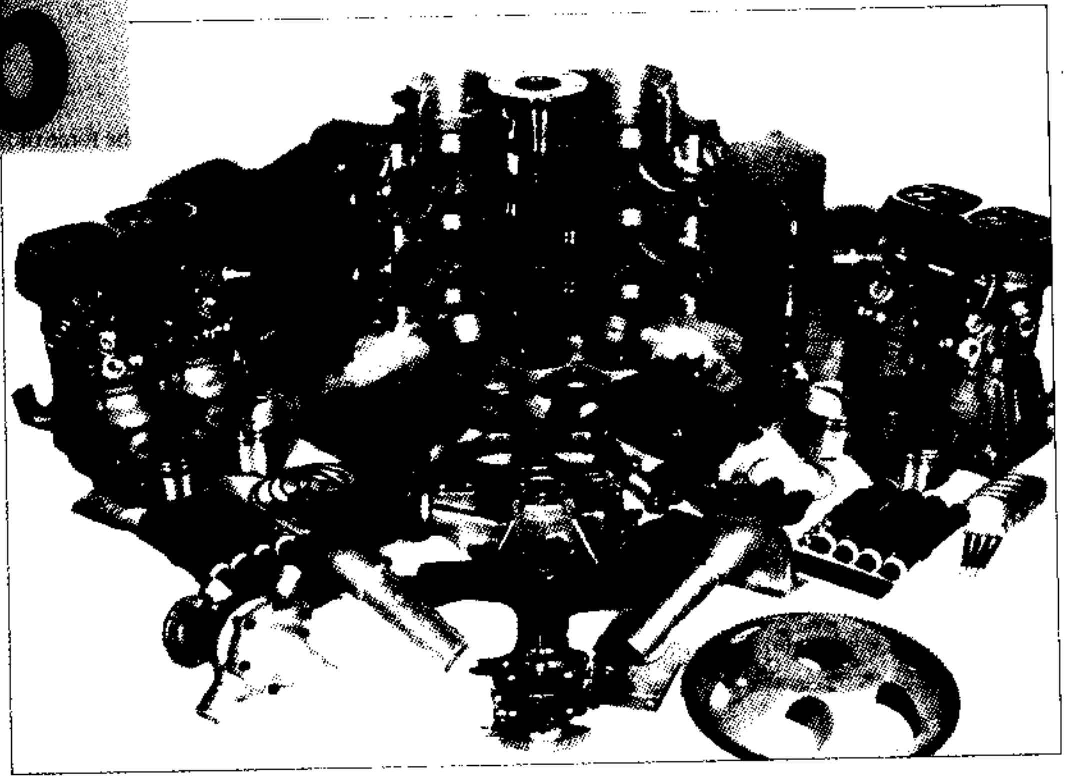


BRAKES, CLUTCHES AND DYNAMOMETERS

6.1 INTRODUCTION

Brakes are the devices that reduce the speed of a moving machine component by absorbing energy. The energy thus absorbed is converted into heat and released into the atmosphere or absorbed in another medium. Clutches are the devices to engage or disengage two rotating machine components as and when desired. Dynamometers, on the other hand, are the devices to measure the power developed by a prime mover. In this chapter, we shall study these devices from the point of view of machine theory.

6.2 BRAKES

Brakes can be classified as follows:

1. Block or shoe brake
2. Band brake
3. Band and block brake
4. Internal expanding shoe brake

6.2.1 Block or Shoe Brake

These brakes may be classified as:

1. Single block or shoe brake

2. Pivoted block or shoe brake
3. Double block or shoe brake

Single block or shoe brake A single block brake is shown in Fig.6.1(a) when the force of friction passes through the fulcrum of the lever. When the angle of contact of the block on the brake drum is small (< 60°), then the normal pressure between the block and the drum can be assumed to be uniform.

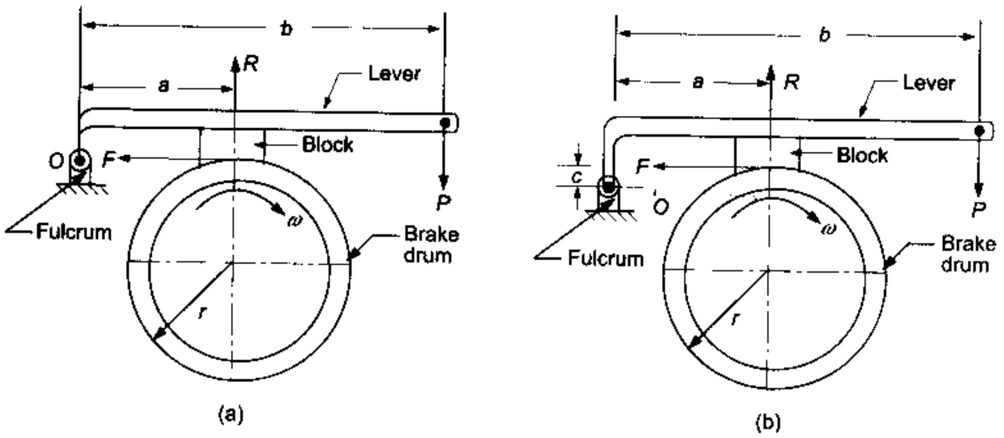


Fig.6.1 Block brake

Normal force between the block and drum, $R = \frac{Pb}{a}$

Tangential braking or frictional force on the drum, $F = \mu R = \frac{\mu Pb}{a}$

Braking torque, $$T_b = Fr = \frac{\mu Pbr}{a} \quad (6.1)$$

where r is the radius of the drum.

When the brake drum is moving on the rails or road with speed v and the braking distance is s, then

Work done against friction $= F \cdot s = \frac{\mu Pb \cdot s}{a}$

Kinetic energy lost $= \frac{1}{2} \cdot mv^2 + \frac{1}{2} \cdot I\omega^2$

where m = mass of brake drum

I = moment of inertia of brake drum

ω = angular speed of drum

For the conservation of energy, we have

$$\frac{\mu Pb \cdot s}{a} = \frac{1}{2} \cdot mv^2 + \frac{1}{2} \cdot I\omega^2 \quad (6.2)$$

1. If the frictional force F is above the lever fulcrum by a distance c, as shown in Fig.6.1(b), then

$$\begin{aligned} P \cdot b &= R \cdot a + F \cdot c \\ &= R \cdot a + \mu R \cdot c \\ &= R(a + \mu c) \end{aligned}$$

or
$$R = \frac{Pb}{a + \mu c} \tag{6.3}$$

We find that the frictional force helps in applying the brake. Such a brake is called *self energizing* brake.

2. If the fulcrum is above the frictional force F by an amount c, then

$$\begin{aligned} P \cdot b + F \cdot c &= R \cdot a \\ P \cdot b &= R(a - \mu c) \end{aligned}$$

or
$$R = \frac{P \cdot b}{a - \mu c} \tag{6.4}$$

If $a \leq \mu c$, then P will be zero or negative, i.e. no external force will be required to apply the brake. Such a brake is called *self-locking* type of brake.

Pivoted shoe brake When the angle subtended by the shoe at the drum centre is more than 60° then the normal pressure is lesser at the sides than at the centre. In such a case (Fig.6.2), consider an element of the block between ϕ and $\phi + d\phi$.

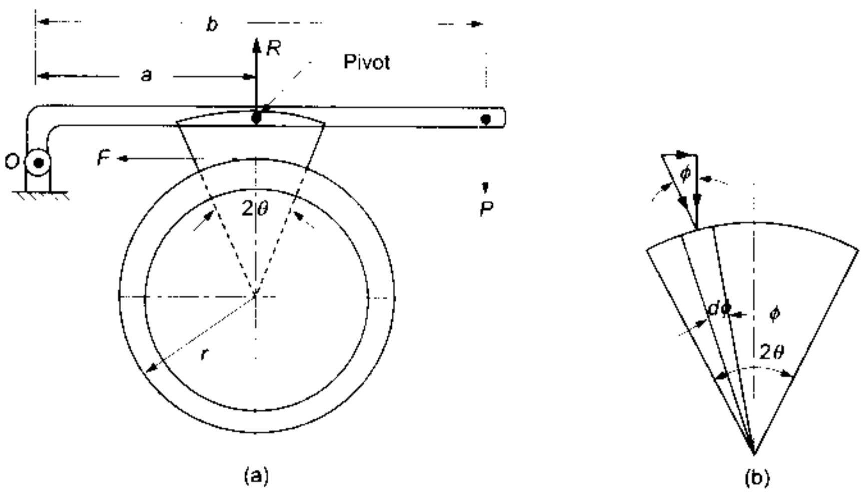


Fig.6.2 Pivoted shoe brake

Let b = width of the drum

r = radius of the drum

p = normal pressure between the block and the drum

Area of the drum element,	$dA = br\, d\phi$
Normal force on the drum,	$dR = pbr\, d\phi$
Vertical component of normal force	$= pbr\, d\phi \cos\phi$
Total normal force,	$R = br \int_{-\theta}^{+\theta} p \cos\phi\, d\phi$
Frictional force on element of the block	$= \mu pbr\, d\phi$
Resisting torque on the drum,	$dT = \mu pbr^2\, d\phi$
Total torque,	$T = \mu br^2 \int_{-\theta}^{+\theta} p\, d\phi$

Normal wear is proportional to the product of normal pressure and rubbing velocity. The component of wear in the direction of applied force P is proportional to $\cos\phi$. Hence, normal pressure is also proportional to $\cos\phi$.

Let
$$p = k\cos\phi$$

where k is the constant of proportionality. Then

$$R = brk \int_{-\theta}^{+\theta} \cos^2\phi\, d\phi$$

$$= \frac{brk(2\theta + \sin 2\theta)}{4}$$

$$T = \mu bkr^2 \int_{-\theta}^{+\theta} \cos\phi\, d\phi$$

$$= 2\mu bkr^2 \sin\theta$$

Eliminating k, we get

$$T = \frac{4\mu R R_r \sin\theta}{2\theta + \sin 2\theta} \qquad (6.5)$$

Equivalent coefficient of friction,

$$\mu_e = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

Double-shoe brake A double-shoe brake is shown in Fig.6.3. It consists of two brake shoes applied at the opposite sides of the brake drum which more or less eliminate the unbalanced force on the shaft due to normal reaction. Frictional or braking torque is given by,

$$T_b = (F_l + F_r)r \qquad (6.6)$$

where F_l and F_r are the frictional forces on the left and right side shoes, respectively, and r is the radius of the brake drum.

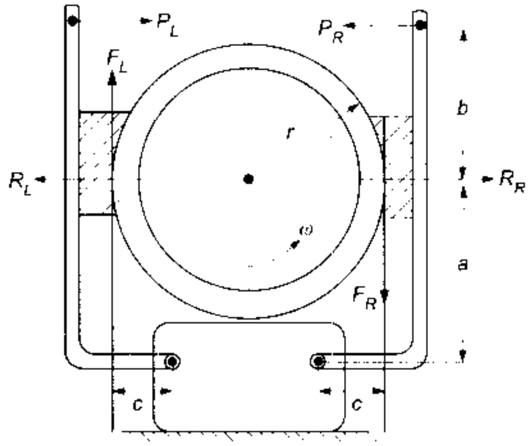


Fig.6.3 Double-shoe brake

Assuming frictional forces passing through the fulcrums of the levers, we have

$$F_l = \mu R_l = \frac{\mu P_l(a+b)}{a}$$

$$F_r = \mu R_r = \frac{\mu P_r(a+b)}{a}$$

When the frictional force is not passing through the fulcrums, then

$$P_l(a+b) + F_l c = R_l a$$

or

$$P_l(a+b) + \mu R_l c = R_l a$$

or

$$R_l = \frac{P_l(a+b)}{a - \mu c}$$

$$F_l = \frac{\mu P_l(a+b)}{a - \mu c}$$

and

$$P_r(a+b) = R_r a + F_r c = R_r a + \mu c$$

or

$$R_r = \frac{P_r(a+b)}{a + \mu c}$$

$$F_r = \frac{\mu P_r(a+b)}{a + \mu c}$$

6.2.2 Band Brake

The band brakes may be classified as: (a) Simple-band brake, and (b) Differential-band brake.

Simple band brake A simple band brake is shown in Fig.6.4(a). Let T_1 and T_2 be the tensions on the tight and slack sides, respectively. Taking moments about the fulcrum, we have

$$Pb = T_1 a, \text{ for counter-clockwise rotation of drum}$$
$$= T_2 a, \text{ for clockwise rotation of drum}$$

Also
$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

Braking torque on the drum, $T_b = (T_1 - T_2)r$ (6.7)

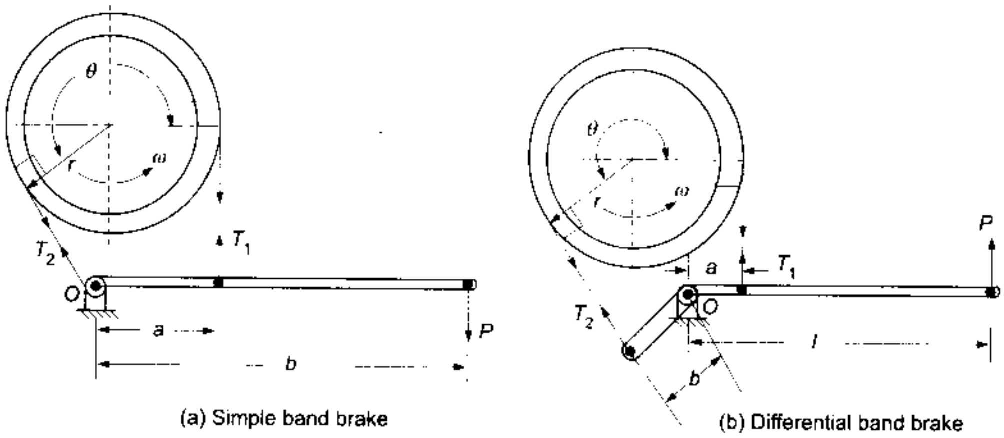


Fig.6.4 Band brake

Differential band brake The differential band brake is shown in Fig.6.4(b). Taking moments about the fulcrum, we have

$$Pl + T_1 a = T_2 b, \text{ for counter-clockwise rotation of drum}$$

or
$$Pl = T_2 b - T_1 a \tag{6.8a}$$

$$Pl + T_2 a = T_1 b, \text{ for clockwise rotation of drum}$$

or
$$Pl = T_1 b - T_2 a \tag{6.8b}$$

For a self-locking brake, $P \leq 0$, therefore, for counter-clockwise rotation,

$$\frac{T_1}{T_2} \geq \frac{b}{a} \tag{6.9a}$$

and for clockwise rotation,

$$\frac{T_1}{T_2} \leq \frac{a}{b} \tag{6.9b}$$

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

6.2.3 Band and Block Brake

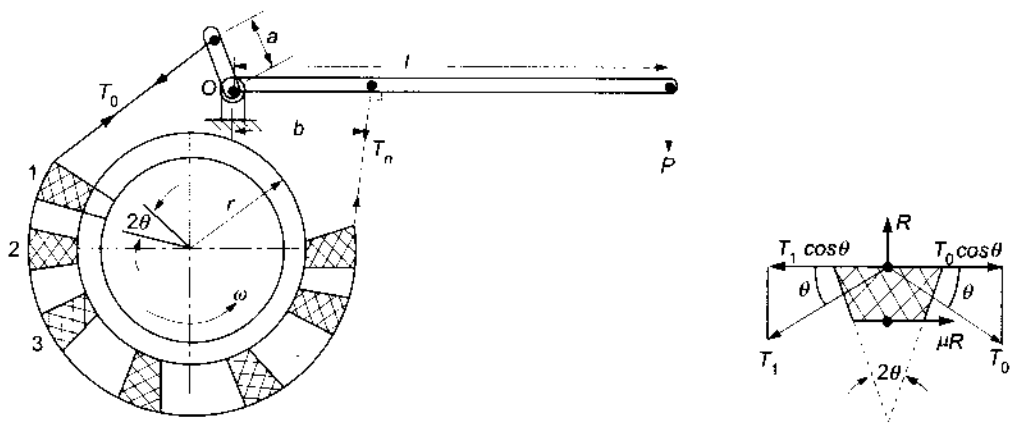
The band and block brake is shown in Fig.6.5(a).

Let T_o = tight side tension in the band on the first block

T_1 = slack side tension in the band on the first block

T_n = slack side tension in the band on the nth block

2θ = angle subtended by the block at the drum centre



(a)

(b) Forces on block 1

Fig.6.5 Block and band brake

The forces acting on the first block are shown in Fig.6.5(b). Resolving the forces horizontally and vertically, we get

$$(T_1 - T_o)\cos\theta = \mu R$$

$$(T_1 + T_o)\sin\theta = R$$

or
$$\frac{T_1 - T_o}{T_1 + T_o}\cot\theta = \mu$$

or
$$\frac{T_1}{T_o} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

Similarly
$$\frac{T_2}{T_1} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

$$\frac{T_n}{T_{n-1}} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

Hence
$$\frac{T_n}{T_o} = \left[\frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}\right]^n \qquad (6.10)$$

Braking torque,
$$T_b = (T_o - T_n)\,r \qquad (6.11)$$

6.2.4 Internal Expanding Shoe Brake

The internal expanding shoe brake is shown in Fig.6.6. The shoes are pinned at O_1 and O_2. The shoes are kept in non-braking position by the spring.

The brakes are applied when the cam is pressed down, in the case of a mechanically operated brake, or when the shoes are pressed on the brake drum, in the case of a hydraulically operated brake.

Consider a small element BC of the shoe between θ and $\theta + d\theta$.

Let r = radius of the brake drum

b = width of the brake lining

p = normal pressure between the shoe and the drum

p_{max} = maximum intensity of normal pressure

F_1 = force exerted by the cam on the leading shoe

F_2 = force exerted by the cam on the trailing shoe

a = distance between the fulcrum O_1 and O.

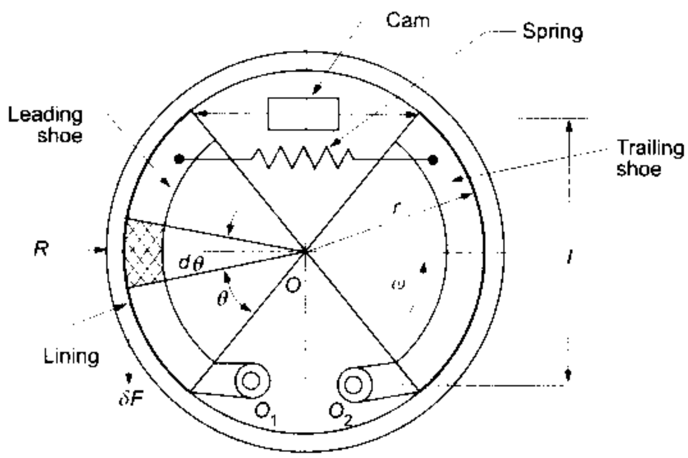


Fig.6.6 Internal expanding shoe brake

It is assumed that the pressure distribution on the shoe is nearly uniform. The shoe turns about point O_1. Therefore, the rate of wear of the shoe lining at B will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OB, i.e. O_1A.

Now $$O_1A = OO_1 \sin\theta = a \sin\theta$$

Normal pressure at B,

$$p \propto \sin\theta$$

or $$p = p_{max} \sin\theta$$

Normal force acting on the element,

$$\delta R = \text{Normal pressure} \times \text{Area of the element}$$
$$= p \cdot br\delta\theta = p_{max} \sin\theta \cdot br\delta\theta$$

Friction force on the element,

$$\delta F = \mu \cdot \delta R = \mu \cdot p_{max} \cdot \sin\theta \cdot br\delta\theta$$

Braking torque due to the element about O,

$$\delta T_b = \delta F \cdot r = \mu p_{max} br^2 \sin\theta \cdot \delta\theta$$

Total braking torque about O,

$$T_b = \mu p_{\max} br^2 \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \mu p_{\max} br^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of the normal force δR about the fulcrum O_1,

$$\delta M_n = \delta R \cdot O_1A = \delta R \cdot OO_1 \sin\theta = \delta R \cdot a \sin\theta$$

Total moment of normal force about the fulcrum O_1,

$$M_n = p_{\max} \cdot b \cdot r \cdot a \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta$$

$$= p_{\max} \cdot b \cdot r \cdot a \int_{\theta_1}^{\theta_2} \left[\frac{1 - \cos 2\theta}{2}\right] d\theta$$

$$= \frac{(p_{\max} \cdot b \cdot r \cdot a)}{2} \cdot \left[\frac{\theta - \sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{(p_{\max} \cdot b \cdot r \cdot a)}{2} \cdot \left[(\theta_2 - \theta_1) + \left(\frac{1}{2}\right) \cdot (\sin 2\theta_1 - \sin 2\theta_2)\right] \tag{6.12}$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_f = \delta F \cdot AB = \delta F(r - a\cos\theta)$$

$$= \mu p_{\max} \sin\theta \cdot br\delta\theta \cdot (r - a\cos\theta)$$

$$= \mu p_{\max} \cdot br(r\sin\theta - a\sin\theta\cos\theta)\delta\theta$$

$$= \mu p_{\max} \cdot br\left(r\sin\theta - \frac{a\sin 2\theta}{2}\right)\delta\theta$$

Total moment of the frictional force about the fulcrum O_1,

$$M_f = \mu p_{\max} \cdot br \int_{\theta_1}^{\theta_2} \left[r\sin\theta - \frac{a\sin 2\theta}{2}\right] d\theta$$

$$= \mu p_{\max} \cdot br \left[-r\cos\theta + \frac{a\cos 2\theta}{4}\right]_{\theta_1}^{\theta_2}$$

$$= \mu p_{\max} \cdot br \left[r(\cos\theta_1 - \cos\theta_2) + \left(\frac{a}{4}\right)(\cos 2\theta_2 - \cos 2\theta_1)\right] \tag{6.13}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \cdot l = M_n - M_f \tag{6.14}$$

and for the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \cdot l = M_n + M_f \tag{6.15}$$

6.2.5 Braking of a Vehicle

Consider a vehicle going up an inclined plane with acceleration a, as shown in Fig.6.7. To stop the vehicle, let brakes be applied to all the four wheels.

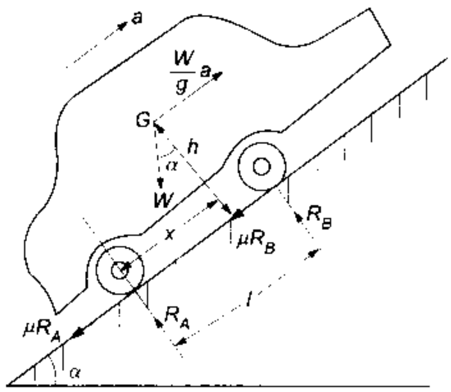


Fig.6.7 Brakes applied to a vehicle going up the inclined plane

Let $F_A = \mu R_A$ = braking force applied at the rear wheels

$F_B = \mu R_B$ = braking force applied at the front wheels

R_A, R_B = normal reactions at A and B

W = weight of the vehicle

h = height of the C.G. of the vehicle from the ground level

l = wheel base (6.16)

Resolving the forces parallel to the plane, we have

$$\mu (R_A + R_B) + W \sin\alpha = \frac{W}{g} \cdot a \tag{6.17}$$

Resolving the forces perpendicular to the plane, we have

$$R_A + R_B = W \cos\alpha \tag{6.18}$$

From (6.18), we get

$$R_B = W \cos\alpha - R_A$$

Taking moments about G, we have

$$\begin{aligned}
\mu (R_A + R_B) h + R_A \cdot x &= (W \cos\alpha - R_A)(l - x) \\
\mu W h \cos\alpha + R_A(x + l - x) &= W(l - x) \cos\alpha \\
R_A &= W \cos\alpha \left[\frac{l - x - \mu h}{l}\right] \\
R_B &= W \cos\alpha \left[\frac{l - (l - x - \mu h)}{l}\right] \\
&= W \cos\alpha \left[\frac{x + \mu h}{l}\right]
\end{aligned}$$

From (6.17), we have

$$\mu W \cos\alpha + W \sin\alpha = \frac{W}{g} \cdot a$$

or

$$a = g(\mu \cos\alpha + \sin\alpha) \tag{6.19}$$

(a) When the vehicle moves on a level track, then $\alpha = 0$, and

$$a = \mu g \tag{6.20}$$

(b) When the vehicle moves down, then

$$a = g(\mu \cos\alpha - \sin\alpha) \tag{6.21}$$

(c) When brakes are applied to rear wheel only, then

$$a = \frac{\mu g(l - x)\cos\alpha}{l + \mu h} \pm g \sin\alpha \tag{6.22}$$

Use the positive sign for going up and the negative sign for going down the plane.
On a level track,

$$a = \frac{\mu g(l - x)}{l + \mu h} \tag{6.23}$$

(d) When brakes are applied to front wheels only, then

$$a = \frac{\mu g x \cos\alpha}{l - \mu h} \pm g \sin\alpha \tag{6.24}$$

Use the positive sign for going up the plane and the negative sign for going down.
On a level track,

$$a = \frac{\mu g x}{l - \mu h} \tag{6.25}$$

Example 6.1

A bicycle and rider of mass 120 kg are travelling at a speed of 15 km/h on a level road. The rider applies brake to the rear wheel which is 0.9 m in diameter. How far will the bicycle travel before it comes to rest? The pressure applied on the brake is 100 N and coefficient of friction between the brake and the cycle rim is 0.05. Assume that no other resistance is acting on the bicycle.

■ Solution

Frictional for, $F = \mu R = 0.05 \times 100 = 5$ N

Let s = be the distance travelled after the bicycle comes to rest, in metres

$$\text{Work done} = F \times s = 5 \cdot s \text{ Nm}$$

Kinetic energy of the wheel, neglecting rotational energy,

$$= \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 120 \times \left(\frac{15 \times 1000}{3600}\right)^2 = 1041.67 \text{ Nm}$$

Hence,

$$5s = 1041.67$$

$$s = 208.33 \text{ m}$$

Example 6.2

A double-shoe brake (Fig.6.8) is capable of absorbing a torque of 1500 N m. The diameter of the brake drum is 300 mm and the angle of contact for each shoe is 90°. The coefficient of friction between the brake drum and the lining is 0.35. Find (a) the spring force necessary to set the brake and (b) width of the brake shoes. The bearing pressure on the lining material is not to exceed 0.25 MPa.

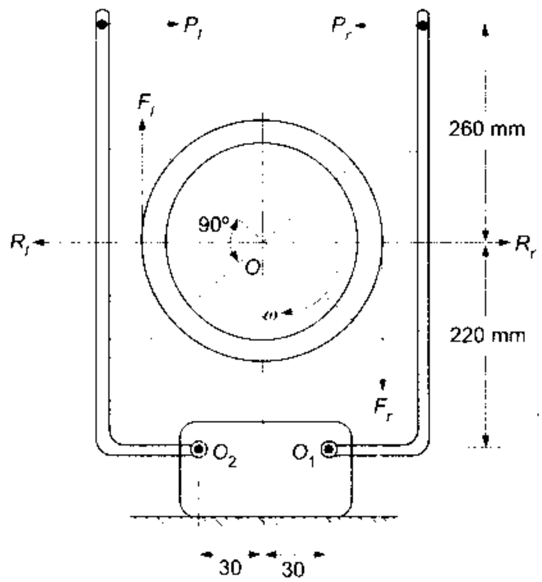


Fig.6.8 Double-shoe brake mechanism

■ **Solution**

(a) Let P be the spring force to set the brake. Since the angle of contact is greater than 60°, therefore, equivalent coefficient of friction,

$$\mu_e = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$
$$= \frac{4 \times 0.35 \times \sin 45°}{2 \times \frac{\pi}{4} + \sin \frac{\pi}{2}}$$
$$= 0.385$$

Taking moments about the fulcrum O_1, we have

$$P_r \times 480 = R_r \times 220 + F_r \times (150 - 30)$$
$$= \left(\frac{F_r}{0.385}\right) \times 220 + F_r \times 120 = 691.428 F_r$$

or

$$F_r = 0.694\ P_r$$

Now taking moments about O_2, we have

$$P_l \times 480 + F_l \times (150 - 30) = R_l \times 220 = \left(\frac{F_l}{0.385}\right) \times 220$$

$$F_l = 1.0633\ P_l$$

Let the spring force, $P_l = P_r = P$

Torque capacity of the brake, $T_b = (F_l - F_r)\,r$

$$1500 = (1.0633 + 0.694)P \times 0.150$$

$$P = 5690.5 \text{ N}$$

(b) Let b be the width of the brake shoes in mm.

Projected bearing area for one shoe, $A_b = b \cdot 2r \sin\theta$

$$= b(2 \times 150 \sin 45°) = 212.1\, b \text{ mm}^2$$

$$R_r = \frac{F_r}{0.385} = \frac{0.694 \times 5690.5}{0.385} = 10257.7 \text{ N}$$

$$R_l = \frac{F_l}{0.385} = \frac{1.0633 \times 5690.5}{0.385} = 15716.1 \text{ N}$$

The normal force is maximum on the left-hand side shoe.

$$0.25 = \frac{15716.1}{212.1\, b}$$

or $b = 296.4$ mm

Example 6.3

A simple band brake, as shown in Fig.6.9, is used on a shaft carrying a flywheel of mass 450 kg. The radius of gyration of the flywheel is 500 mm, and runs at 320 rpm. The coefficient of friction is 0.2 and the brake drum diameter is 250 mm. Find (a) torque applied due to a hand load of 120 N, (b) the number of turns of the wheel before it is brought to rest and (c) the time required to bring it to rest from the moment of application of the brake.

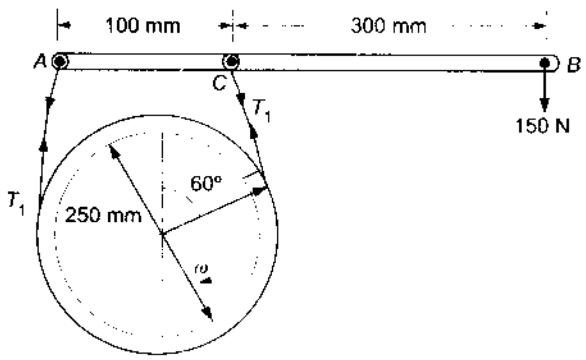


Fig.6.9 Simple band brake mechanism

■ Solution

(a) Angle of contact, $\theta = 210° = 3.6652$ rad

$$\frac{T_1}{T_2} = \exp(\mu\theta) = \exp(0.2 \times 3.6652) = 2.08$$

Taking moments about the fulcrum C, we have

$$T_2 \times 100 = 150 \times 300$$

$$T_2 = 450 \text{ N}$$

$$T_1 = 936 \text{ N}$$

Torque applied, $T_b = (T_1 - T_2)\, r = (936 - 450) \times 0.125$

$$= 60.75 \text{ Nm}$$

(b) Rotational kinetic energy of the wheel $= \frac{1}{2} \cdot I\omega^2$

$$= \frac{1}{2} \cdot mK^2 \cdot \omega^2$$

$$= \frac{1}{2} \times 450 \times 0.5^2 \times \left(2\pi \times \frac{320}{60}\right)^2$$

$$= 63165.5 \text{ Nm}$$

Energy used to overcome the braking torque $= 2\pi n T_b$

$$= 2\pi \times n \times 60.75$$

$$= 381.7\, n \text{ Nm}$$

$$= 63165.5$$

or $n = 165$

(c) Time required to bring the wheel to rest $= \dfrac{n}{\left(\frac{N}{2}\right)}$

$$= \frac{2 \times 165}{320} = 1.031 \text{ min or } 61.87 \text{ s}$$

Example 6.4

A differential band brake shown in Fig.6.10, has an angle of contact of 225°. The band has a lining whose coefficient of friction is 0.3 and the drum diameter is 400 mm. The brake is to sustain a torque of 375 N m. Find (a) the necessary force for the clockwise and counter-clockwise rotation of the drum and (b) the value of OA for the brake to be self locking, when the drum rotates clockwise.

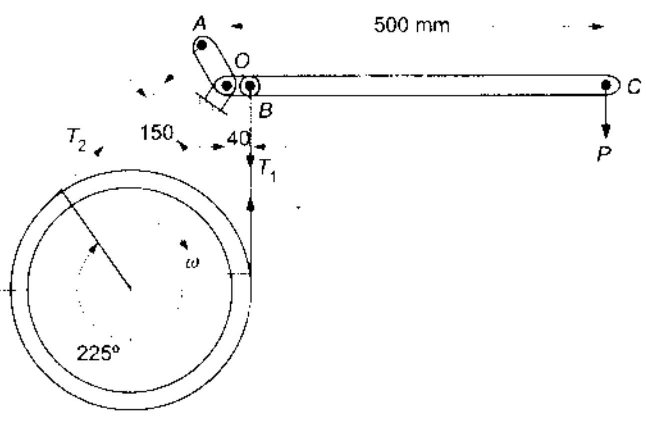


Fig.6.10 Differential band brake mechanism

Solution

1. Force required

(a) Clockwise rotation of the drum

$$\frac{T_1}{T_2} = \exp(\mu\theta) = \exp\left(\frac{0.3 \times \pi \times 225}{180}\right) = 3.248$$

$$T_b = (T_1 - T_2)\, r$$

$$375 = (T_1 - T_2) \times 0.2$$

$$T_1 - T_2 = 1875 \text{ N}$$

$$T_1 = 2709 \text{ N}, \qquad T_2 = 834 \text{ N}$$

Taking moments about the fulcrum O, we have

$$P \times 500 + T_1 \times 40 = T_2 \times 150$$

$$P \times 500 + 2709 \times 40 = 834 \times 150, \quad P = 46.7 \text{ N}$$

(b) Counter-clockwise rotation of the drum

Taking moments about O, we have

$$P \times 500 + T_2 \times 40 = T_1 \times 150$$

$$P \times 500 + 834 \times 40 = 2790 \times 150, \quad P = 903.7 \text{ N}$$

2. For the brake to be self-locking, $P = 0$. For clockwise rotation of the drum,

$$T_1 \times 40 = T_2 \times OA$$

$$OA = \frac{2709 \times 40}{834} = 129.92 \text{ mm}$$

Example 6.5

A band and block brake, with 15 blocks, each of which subtends an angle of 15°, is applied to a drum of 1 m diameter, as shown in Fig.6.11. The drum and the flywheel mounted on the same shaft has a mass of 1500 kg and a combined radius of gyration of 500 mm. Find (a) maximum braking torque, (b) angular retardation of the drum and (c) time taken by the system to come to rest from the rated speed of 380 rpm. The coefficient of friction between the drum and the blocks can be taken as 0.25.

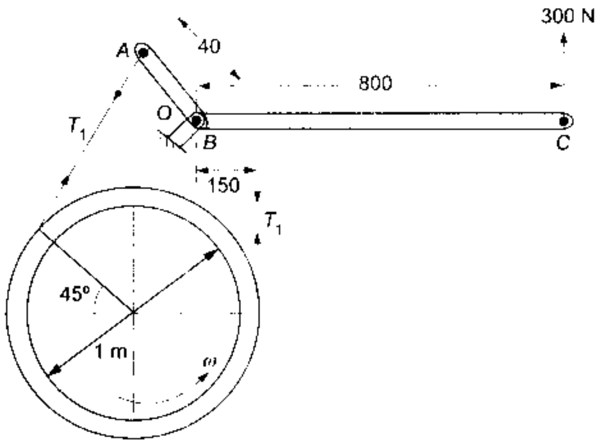


Fig.6.11 Band and block brake mechanism

Solution

(a) The braking torque will be maximum when the drum rotates counter-clockwise and the force P is upwards. Taking moments about O, we have

$$300 \times 800 + T_1 \times 40 = T_2 \times 150$$

$$15T_2 - 4T_1 = 24000$$

$$\frac{T_1}{T_2} = \left[\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right]^n$$

$$= \left[\frac{1 + 0.25 \times \tan 7.5°}{1 - 0.25 \times \tan 7.5°}\right]^{15} = 2.685$$

$$15T_2 - 4 \times 2.685T_2 = 24000$$

$$4.26T_2 = 24000$$

$$T_2 = 5633.8 \text{ N}$$

$$T_1 = 15126.76 \text{ N}$$

Braking torque,

$$T_b = (T_1 - T_2)r = (15126.76 - 5633.8) \times 0.5$$

$$= 4744 \text{ Nm}$$

(b) Let α be the angular retardation of the drum

$$T_b = I\alpha = mK^2 \cdot \alpha$$

$$4744 = 1500 \times (0.5)^2 \cdot \alpha$$

$$\alpha = 12.65 \text{ rad/s}^2$$

(c) Let t be the time taken to come to rest

Initial angular speed,

$$\omega_1 = \frac{2\pi \times 380}{60} = 39.793 \text{ rad/s}$$

Final angular speed,

$$\omega_2 = 0$$

$$\omega_2 = \omega_1 - \alpha t$$

$$t = \frac{39.793}{12.65} = 3.15 \text{ s}$$

Example 6.6

An external expanding shoe brake is shown in Fig.6.12. The coefficient of friction may be taken as 0.35, and the braking torque required is 25 N m. Calculate the force P required to operate the brake when the drum rotates (a) clockwise and (b) counter-clockwise.

Solution

(a) When drum rotates clockwise,

Total braking torque,

$$T_b = \mu p_{max} \cdot b \cdot r^2 (\cos\theta_1 - \cos\theta_2)$$

$$25 = 0.35 \times p_{max} \times b \times 0.12^2 (\cos 30° - \cos 150°)$$

$$p_{max}\, b = 2863.84$$

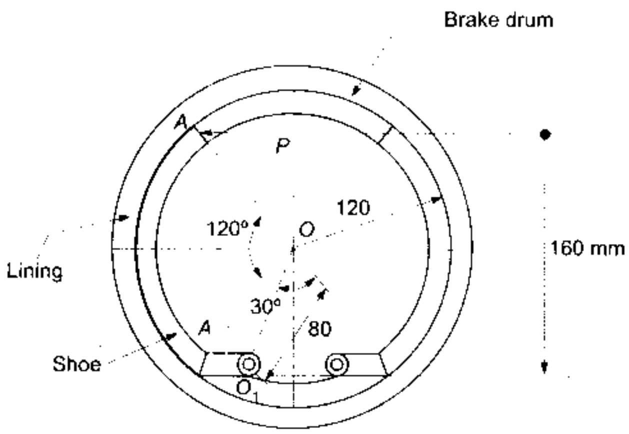


Fig.6.12 External expanding shoe brake mechanism

Total moment of the normal force about the fulcrum O_1.

$$\begin{aligned} M_n &= 0.5 p_{max}\, bra\, [(\theta_2 - \theta_1) + 0.5(\sin 2\theta_1 - \sin 2\theta_2)] \\ &= 0.5 \times 2863.84 \times 0.12 \times 0.08 \left[\frac{(150-30)\times\pi}{180 + 0.5(\sin 60^\circ - \sin 300^\circ)}\right] \\ &= 40.695 \text{ Nm} \end{aligned}$$

Total moment of friction force about the fulcrum O_1.

$$\begin{aligned} M_f &= \mu p_{max}\, br\{r(\cos\theta_1 - \cos\theta_2) + 0.25\, a(\cos 2\theta_2 - \cos 2\theta_1)\} \\ &= 0.35 \times 2863.84 \times 0.12\, [0.12(\cos 30^\circ - \cos 150^\circ) + 0.25 \times 0.08(\cos 300^\circ - \cos 60^\circ)] \\ &= 25 \text{ Nm} \end{aligned}$$

$$\text{Total moment} = M_n + M_f = 65.695 \text{ Nm}$$

Taking moments about O_1, we have

$$\begin{aligned} P \times 0.16 &= 65.695 \\ P &= 410.6 \text{ N} \end{aligned}$$

(b) When drum rotates counter-clockwise, taking moments about O_1, we have

$$\begin{aligned} P \times 0.16 &= M_n - M_f = 15.695 \\ P &= 98.1 \text{ N} \end{aligned}$$

Example 6.7

A vehicle moving on a rough plane inclined at 15° with the horizontal at a speed of 40 km/h has a wheel base of 1.8 m. The centre of gravity of the vehicle is 0.8 m from the rear wheels and 0.9 m above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when the vehicle is moving (a) up the plane, and (b) down the plane.The brakes are applied to all the four wheels and the coefficient of friction is 0.45.

■ **Solution**

(a) For the vehicle moving up the plane,

$$a = g(\mu \cos\alpha - \sin\alpha)$$
$$= 9.81(0.45 \times \cos 15° + \sin 15°)$$
$$= 6.8 \text{ m/s}^2$$

Distance travelled,
$$s = \frac{u^2}{2a} = \frac{\left(\frac{40 \times 1000}{3600}\right)^2}{2 \times 6.8}$$
$$= 9.078 \text{ m}$$

Final velocity of vehicle,
$$v = u + a\,t$$
$$0 = 11.11 - 6.8\,t$$
$$t = 1.63 \text{ s}$$

(b) For the vehicle going down the plane,

$$a = g(\mu \cos\alpha - \sin\alpha)$$
$$= 9.81(0.45 \times \cos 15° - \sin 15°)$$
$$= 1.725 \text{ m/s}^2$$
$$s = \frac{u^2}{2a} = \frac{(11.11)^2}{2 \times 1.725} = 35.78 \text{ m}$$
$$t = \frac{u}{a} = \frac{11.11}{1.725} = 6.44 \text{ s}$$

Example 6.8

The wheel base of a car is 3 m and its centre of gravity is 1.2 m ahead of the rear axle and 0.75 m above the ground level. The coefficient of friction between the wheels and the road is 0.5. Determine the maximum deceleration of the car when it moves on a level if the braking force on all the four wheels is the same and no wheel slip occurs.

■ **Solution**

(a) When slipping is imminent on the rear wheels (Fig.6.13).

$$R_A = W\left[\frac{l - \mu h - x}{l}\right]$$
$$= W\left[\frac{3 - 0.5 \times 0.75 - 1.2}{3}\right] = 0.475\ W \text{ N}$$
$$F_A + F_B = W\frac{a}{g}$$

Now
$$F_A = F_B \quad \text{and} \quad F_A = \mu R_A$$
$$2\mu R_A = W$$

or
$$2 \times 0.5 \times 0.475\ W = W\frac{a}{g}$$
$$a = 4.66 \text{ m/s}^2$$

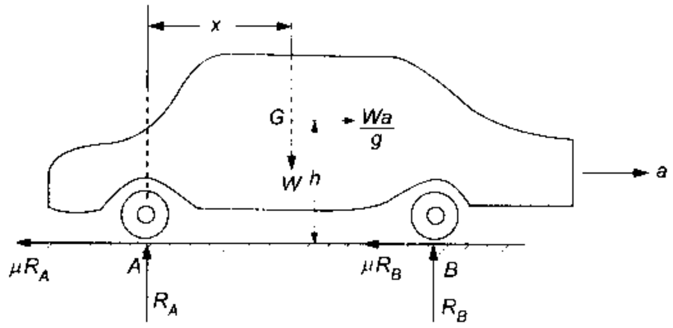


Fig.6.13 Forces on the vehicle

(b) When slipping is imminent on the front wheels.

$$R_B = W\left[\frac{\mu h + x}{l}\right]$$

$$= W\left[\frac{0.5 \times 0.75 + 1.2}{3}\right] = 0.525\ W\ \text{N}$$

$$F_A + F_B = \frac{Wa}{g}$$

Now $\quad F_A = F_B \quad \text{and} \quad F_B = \mu R_B$

$$2\mu R_B = W$$

or $\quad 2 \times 0.5 \times 0.525\ W = \frac{Wa}{g}$

$$a = 5.15\ \text{m/s}^2$$

6.3 CLUTCHES

The friction clutches may be classified as follows:

1. Plate (or disc) clutches
   (a) Single plate clutches
   (b) Multiple plate clutches
2. Cone clutch

6.3.1 Single Plate Clutch

Consider a single plate clutch as shown in Fig.6.14.

Let r_1, r_2 = inner and outer radii of the plate respectively

p = intensity of axial pressure on the plate

W = axial load on the clutch

μ = coefficient of friction between the friction surfaces and the plate

T = torque transmitted by the clutch

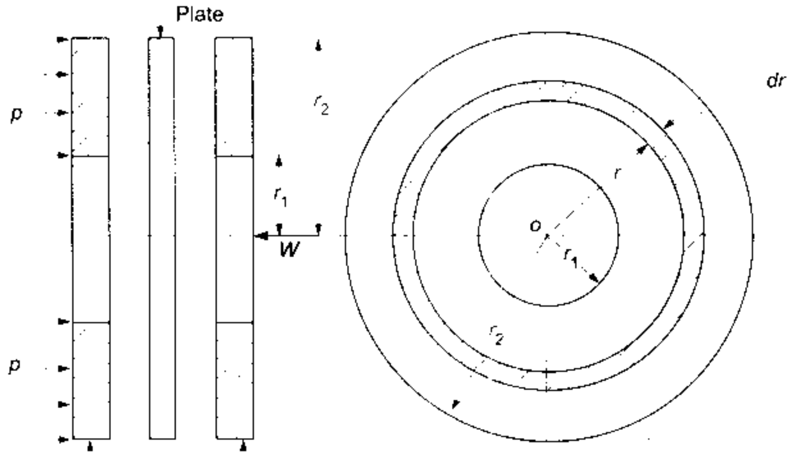


Fig.6.14 Single plate clutch

Consider an elementary ring of the friction surface of radius r and thickness dr.

Contact area of the friction surface, $dA = 2\pi r \cdot dr$

Axial force on the ring, $dW = p \cdot dA$

Frictional force on the ring, $dF = \mu \cdot dW$

Frictional torque on the ring, $dT_f = dF \cdot r = 2\pi\mu \cdot p \cdot r^2 \cdot dr$

(a) Uniform pressure

When the pressure is uniform over the entire area of friction surface, then the intensity of pressure,

$$p = \frac{W}{\pi\left(r_2^2 - r_1^2\right)}$$

Total frictional torque on the frictional surface,

$$T_f = 2\pi\mu\pi\int_{r_1}^{r_2} r^2 dr = \frac{2}{3}\pi\mu p\left(r_2^3 - r_1^3\right)$$

$$= \left(\frac{2}{3}\right) \cdot \mu W\left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad (6.26)$$

Mean radius of friction surface, $$r_m = \left(\frac{2}{3}\right) \cdot \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad (6.27)$$

(b) Uniform wear

For uniform wear, the intensity of pressure varies inversely with the distance, therefore

$$p \cdot r = C$$

or $$p = \frac{C}{r}$$

Force acting on the ring, $$dW = \frac{C}{r} \cdot 2\pi r \cdot dr = 2\pi C \cdot dr$$

Total force acting on the friction surface, $$W = \int_{r_1}^{r_2} 2\pi C \cdot dr = 2\pi C\,(r_2 - r_1)$$

or $$C = \frac{W}{2\pi\,(r_2 - r_1)}$$

Frictional torque on the ring, $$dT_f = 2\pi\mu \cdot C \cdot r \cdot dr$$

Total frictional torque on the friction surface,

$$T_f = 2\pi\mu \cdot C \int_{r_1}^{r_2} r \cdot dr = \pi\mu \cdot C\left(r_2^2 - r_1^2\right)$$

$$= \mu W \frac{(r_1 + r_2)}{2} \quad (6.28)$$

$$r_m = \frac{r_1 + r_2}{2} \quad (6.29)$$

6.3.2 Multiple Plate Clutch

Multiplate friction clutches are used where space is a limitation, as in the case of scooters, etc.

Let n_1 = number of plates on the driving shaft

n_2 = number of plates on the driven shaft

Then, number of pairs of contact surfaces, $$n = n_1 + n_2 - 1$$

Total frictional torque transmitted, $$T = n\mu W r_m$$

6.3.3 Cone Clutch

Consider a cone clutch as shown in Fig.6.15(a).

Let r_1, r_2 = inner and outer radii of the frictional conical surface

p_n = normal pressure between the contact surfaces

b = width of the conical surfaces

μ = coefficient of friction between contact surfaces

α = semi-cone angle.

Consider an elementary ring of the conical surface of radii r and $r + dr$ and of length dl, as shown in Fig.6.15(b).

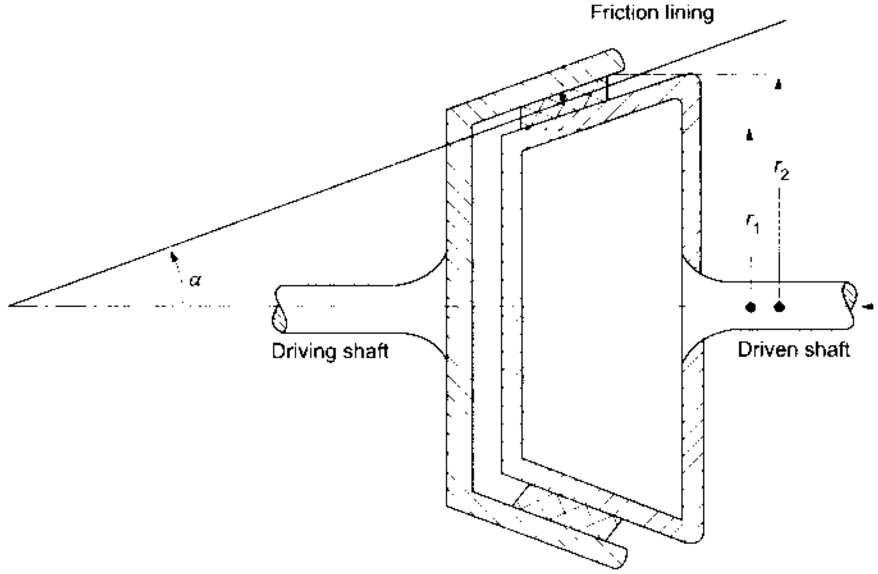
$$dl = dr \cdot \operatorname{cosec}\alpha$$

Area of the ring, $$dA = 2\pi r \cdot dl = 2\pi r \cdot dr \cdot \operatorname{cosec}\alpha$$

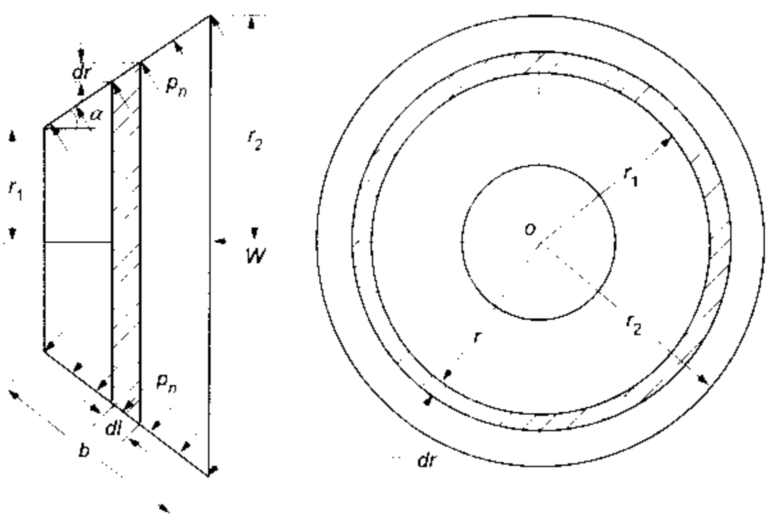
(a) Uniform pressure

Normal load acting on the ring, $$dW_n = p_n \cdot dA$$

Axial load acting on the ring, $$dW = dW_n \cdot \sin\alpha = 2\pi p_n \cdot r \cdot dr$$



(a) Clutch geometry



(b) Forces on cone

Fig.6.15 Cone clutch

Total axial load transmitted to the clutch,

$$W = 2\pi p_n \int_{r_1}^{r_2} r \cdot dr = \pi p_n \left(r_2^2 - r_1^2\right)$$

or

$$p_n = \frac{W}{\pi \left(r_2^2 - r_1^2\right)}$$

Frictional force on the ring, $dF = \mu \cdot dW_n$

Frictional torque, $dT_f = dF \cdot r = 2\pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \cdot r^2 dr$

Total frictional torque,

$$T_f = 2\pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \int_{r_1}^{r_2} r^2 \cdot dr$$

$$= \left(\frac{2}{3}\right) \cdot \pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \left(r_2^3 - r_1^3\right)$$

$$= \left(\frac{2}{3}\right) \cdot \mu W \cdot \text{cosec}\,\alpha \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad (6.30a)$$

$$= \mu W \cdot r_m \,\text{cosec}\,\alpha \quad (6.30b)$$

(b) Uniform wear

For uniform wear, $p_n \cdot r = C$

or $p_n = \dfrac{C}{r}$

Normal load on the ring, $\mathrm{d}W_n = p_n \cdot \mathrm{d}A$

Axial load on the ring, $\mathrm{d}W = \mathrm{d}W_n \cdot \sin\alpha = p_n \cdot 2\pi r \cdot \mathrm{d}r = 2\pi C \cdot \mathrm{d}r$

Total axial load on the clutch,

$$W = 2\pi C \int_{r_1}^{r_2} \mathrm{d}r = 2\pi C\,(r_2 - r_1)$$

or

$$C = \frac{W}{2\pi\,(r_2 - r_1)}$$

Frictional force on the ring, $\mathrm{d}F = \mu \cdot \mathrm{d}W_n$

Frictional torque on the ring,

$$\mathrm{d}T_f = \mathrm{d}F \cdot r = 2\pi\mu p_n \cdot \text{cosec}\,\alpha \cdot r^2 \cdot \mathrm{d}r$$

$$= 2\pi\mu C \cdot \text{cosec}\,\alpha \cdot r \cdot \mathrm{d}r$$

Frictional torque on the clutch,

$$T_f = 2\pi\mu C \cdot \text{cosec}\,\alpha \int_{r_1}^{r_2} r \cdot \mathrm{d}r$$

$$= \pi\mu C \cdot \text{cosec}\,\alpha \left(r_2^2 - r_1^2\right)$$

$$= \mu W \text{cosec}\,\alpha \frac{(r_1 + r_2)}{2}$$

$$= \mu W r_m \text{cosec}\,\alpha \quad (6.31)$$

where $r_m = \dfrac{r_1 + r_2}{2}$ is the mean radius.

The frictional torque can also be written as,

$$T_f = 2\pi\mu \cdot p_n \cdot b \cdot r^2 m \quad (6.32)$$

Axial force required to engage the clutch,

$$W_e = W + \mu W_n \cos\alpha$$

$$= W_n \sin\alpha + \mu W_n \cos\alpha$$

$$= W_n(\sin\alpha + \mu\cos\alpha) \quad (6.33)$$

Example 6.9

A single plate clutch, with both sides effective, has inner and outer diameters of friction surface 250 mm and 350 mm respectively. The maximum intensity of pressure is not to exceed 0.15 MPa. The coefficient of friction is 0.3. Determine the power transmitted by the clutch at a speed of 2400 rpm for (a) uniform wear and for (b) uniform pressure.

■ **Solution**

(a) For uniform wear, $T_f = n \cdot \mu W \cdot r_m$

$$r_m = \frac{250 + 350}{4} = 150 \text{ mm} = 0.15 \text{ m}$$

$$n = 2$$

$$C = pr_1 = 0.15 \times 125 = 18.75 \text{ N/mm}$$

$$W = 2\pi C\,(r_2 - r_1) = 2\pi \times 18.75(175 - 125) = 5890.48 \text{ N}$$

$$T_f = 2 \times 0.3 \times 5890.48 \times 0.15 = 530.14 \text{ Nm}$$

Power transmitted,
$$P = \frac{T_f \omega}{1000} \text{ kW} = 530.14 \times \frac{\left(2\pi \times \frac{2400}{60}\right)}{1000}$$
$$= 133.24 \text{ kW}$$

(b) For uniform pressure,
$$r_m = \left(\frac{2}{3}\right) \cdot \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] = \left(\frac{2}{3}\right) \cdot \left[\frac{175^3 - 125^3}{175^2 - 125^2}\right]$$
$$= 151.4 \text{ mm}$$
$$W = \pi\left(r_2^2 - r_1^2\right)p$$
$$= \pi(175^2 - 125^2) \times 0.15 = 7068.58 \text{ N}$$
$$T_f = n\mu W r_m$$
$$= 2 \times 0.3 \times 7068.58 \times 151.4 \times 10^{-3}$$
$$= 642.1 \text{ Nm}$$

Power transmitted,
$$P = 642.1 \times \frac{\left(2\pi \times \frac{2400}{60}\right)}{1000}$$
$$= 161.38 \text{ kW}$$

Example 6.10

A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The inside and outside diameters of the friction surfaces are 125 mm and 250 mm respectively. Assuming uniform pressure and coefficient of friction equal to 0.3, find the total spring load pressing the plates together to transmit 30 kW at 1500 rpm.

■ **Solution**

Power transmitted,
$$P = \frac{T_f \omega}{1000} \text{ kW}$$

$$30 = \frac{T_f \times \left(2\pi \times \frac{1500}{60}\right)}{1000}$$

$$T_f = \frac{(30 \times 1000 \times 60)}{(2\pi \times 1500)} = 190.986 \text{ Nm}$$

$$r_m = \left(\frac{2}{3}\right)\left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]$$

$$= \left(\frac{2}{3}\right) \cdot \left[\frac{125^3 - 62.5^3}{125^2 - 62.5^2}\right] = 97.2 \text{ mm}$$

$$T_f = n\mu W r_m$$

$$190.986 = 4 \times 0.3 \times W \times 0.0972$$

$$W = 1637.4 \text{ N}$$

Example 6.11

An engine developing 50 kW at 1200 rpm is fitted with a cone clutch. The cone angle is 12° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.25. The normal pressure on the clutch face is not to exceed 0.1 MPa. Determine (a) the axial spring force to engage the clutch, and (b) the face width required.

■ **Solution**

(a)

$$P = \frac{T_f \omega}{1000} \text{ kW}$$

$$50 = \frac{T_f \times \left(2\pi \times \frac{1200}{60}\right)}{1000}$$

$$T_f = 397.9 \text{ Nm}$$

$$= \mu W_n r_m = 0.25 \times W_n \times 0.25$$

$$W_n = 6366.4 \text{ N}$$

Axial spring force required to engage the clutch.

$$W_e = W_n(\sin\alpha + \mu\cos\alpha)$$

$$= 6366.4(\sin 12° + 0.25\cos 12°)$$

$$= 2880.5 \text{ N}$$

(b)

$$W_n = p_n \cdot 2\pi r_m b$$

Face width,

$$b = \frac{2880.5}{0.1 \times 2\pi \times 250} = 18.3 \text{ mm}$$

6.4 DYNAMOMETERS

The two types of dynamometers are:

1. Absorption dynamometers, and 2. Transmission dynamometers.

6.4.1 Absorption Dynamometers

In these type of dynamometers, the entire power produced by the prime mover is absorbed by the frictional resistance of the brake and is transformed into heat, during the process of measurement. The absorption type of dynamometers can be classified as: (a) Prony brake dynamometer and (b) Rope brake dynamometer.

Prony brake dynamometer The prony brake dynamometers is shown in Fig.6.16. It consists of two wooden blocks placed around a pulley fixed to the shaft of the prime mover, whose power is to be measured. The blocks are clamped by means of two bolts and nuts. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its other end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S are provided to limit the motion of the lever.

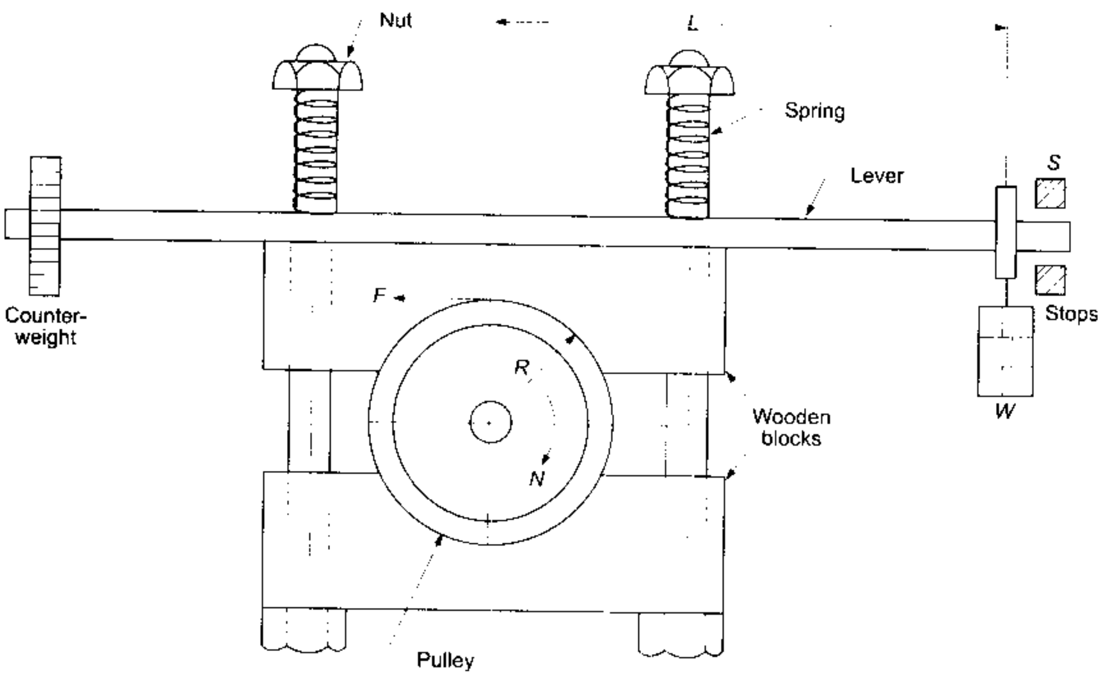


Fig.6.16 Prony brake dynamometer

When the brake is to be put in operation, the long end of the lever is loaded with suitable weight W and the nuts are tightened until the prime mover shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

Moment of the frictional resistance, $T_f = WL = FR$

Work done per minute $= 2\pi N T_f$

Brake power,
$$BP = \frac{2\pi N T_f}{60 \times 1000} = \frac{2\pi N W L}{60 \times 1000} \text{ kW} \qquad (6.34)$$

Rope brake dynamometers The rope brake dynamometers is shown in Fig.6.17. It consists of one or more ropes wound around the flywheel or rim of the pulley, fixed rigidly to the shaft of the prime mover. The upper end of the ropes is attached to a spring balance while the lower end is kept in position by applying a dead weight. In order to prevent the slipping of the ropes over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

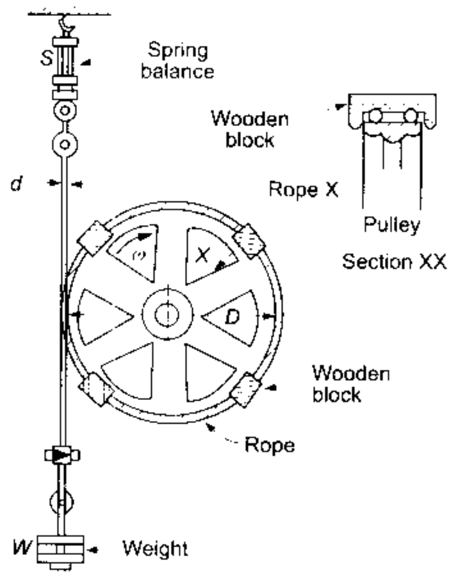


Fig.6.17 Rope brake dynamometer

During operation of the brake, the prime mover is made to run at a constant speed. The frictional torque due to the ropes must be equal to the torque being transmitted by the prime mover.

Let W = dead load on the rope

S = spring balance reading

D = diameter of the pulley

d = diameter of the rope

N = speed of the pulley

Work done per minute $= (W - S)\,\pi\,(D + d)N$

Brake power, $$BP = \frac{(W - S)\,\pi\,(D + d)N}{60 \times 1000}\text{ kW} \qquad (6.35)$$

6.4.2 Transmission Dynamometers

In these type of dynamometers, the energy is used for doing work. The power developed by the prime mover is transmitted through the dynamometer to some other machine where the power is suitably measured. This type of dynamometer can be classified as follows:

1. Epicyclic train dynamometer
2. Belt transmission dynamometer
3. Torsion dynamometer.

Epicyclic train dynamometer The epicyclic train dynamometer is shown in Fig.6.18. It consists of a simple epicyclic train of gears: a spur gear, an annular gear and a pinion. The spur gear is keyed to the engine shaft and rotates in counter-clockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight w is placed at the smaller end of the lever in order to keep it in position.

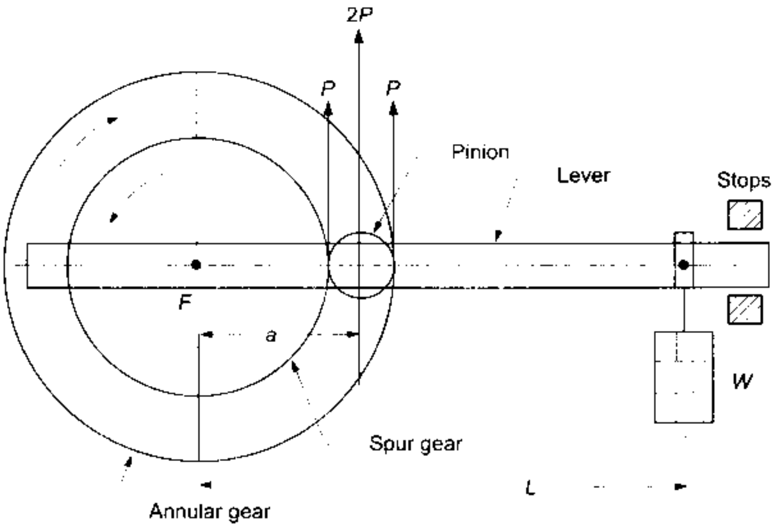


Fig.6.18 Epicyclic gear train dynamometer

Let P be the force between the pinion and the spur gear and the annular gear. Then the total upward force on the lever through the axis of the pinion is $2P$. This force is balanced by a dead weight W at the other end of the lever. The stops control the movement of the lever.

For the equilibrium of the lever, taking moments about the fulcrum, we have

$$2P \cdot a = WL$$

or

$$P = \frac{WL}{2a}$$

Torque transmitted, $T = PR$

where R = pitch circle radius of the spur gear.

Power transmitted,

$$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi NPR}{60 \times 1000} \text{ kW}$$

Belt transmission dynamometer A belt transmission dynamometer, as shown in Fig.6.19, consists of a driving pulley A, rigidly fixed to the shaft of the prime mover. There is another driven pulley B mounted on another shaft to which the power from pulley A is transmitted. The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D, which are mounted on the lever, pivoted at E. The lever carries a dead weight W at the one end and at the other end a balancing weight is attached. The total force acting on the pulley D is $2T_1$ and on the pulley $2T_2$. Taking moments about the fulcrum E, we have

$$2T_1 L = 2T_2 a + WL$$

or

$$T_1 - T_2 = \frac{WL}{2a}$$

Brake power developed,

$$BP = \frac{\pi DN(T_1 - T_2)}{60 \times 1000} \text{ kW} \tag{6.36}$$

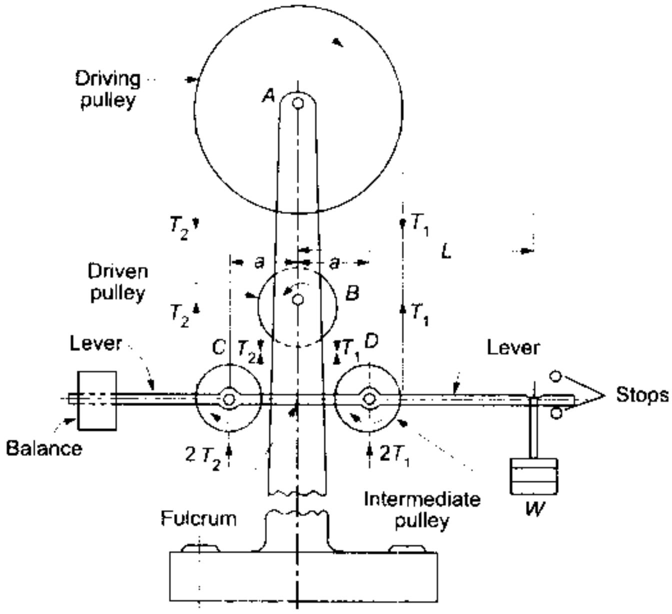


Fig.6.19 Belt transmission dynamometer

Torsion dynamometer A torsion dynamometer is used to measure large power developed by a turbine or marine engines. The torque developed by a shaft of diameter d, length l, and modulus of rigidity G, is

$$T = \left(\frac{GJ}{l}\right)\theta = k\theta$$

where θ = angle of twist of the shaft.

Therefore, the torque acting on the shaft is proportional to the angle of twist.

By measuring the angle of twist, the power developed by the machine can be measured.

Power developed,
$$P = \frac{2\pi NT}{60 \times 1000} \text{ kW} \tag{6.37}$$

A large number of torsion dynamometers are used to measure the angle of twist. We describe below the flash light dynamometer.

Flash light dynamometer The principle of the flash light dynamometer is shown in Fig.6.20. It consists of two discs *A* and *B* fixed on a shaft at a convenient distance apart. Each disc has a small radial slot and these two slots are in the same line when no power is transmitted and there is no torque on the shaft. A bright electric lamp behind the disc *A* is fixed on the bearing of the shaft. This lamp is masked having a slot directly opposite to the slot of disc *A*. At every revolution of the shaft, a flash of light is projected through the slot in the disc *A* towards the disc *B* in a direction parallel to the shaft. An eye piece is fitted behind the disc *B* on the shaft bearing and is capable of slight circumferential adjustment.

When the shaft does not transmit any torque, a flash of light may be seen after every revolution of the shaft, as the positions of the slot do not change relative to one another, as shown in Fig.6.20(b). When the torque is transmitted, the shaft twists and the slot in the disc *B* changes its position, though the slots in the lamp, disc *A*, and eye piece are still in line. Due to this, the light does not reach the eye piece, as shown in Fig.6.20(c). If the eye piece is now moved round by an amount equal to the lag of the disc *B*, then the slot in the eye piece will be opposite to the slot in disc *B*, and hence the eye piece will receive flash of light. The eye piece is moved by operating a micrometer spindle and by means of scale and Vernier, the angle of twist may be measured upto $1/100^{th}$ of a degree. For the measurement of variable torque, the discs *A* and *B* should be perforated with slots arranged in the form of a spiral.

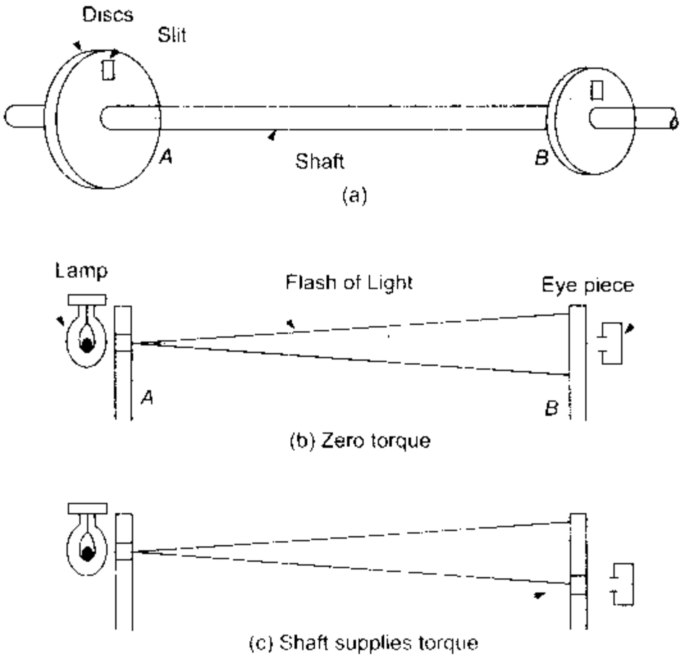


Fig.6.20 Flash-Light dynamometer

Exercises

1 In a single block brake, the drum diameter is 300 mm, the angle of contact is 90°, and the coefficient of friction between the lining and the drum is 0.30. If the operating force is 400 N, applied at the end of a lever 400 mm long, determine the torque transmitted by the brake. The distance of the fulcrum from the centre of the brake drum is 200 mm and assume that the force of friction passes through the fulcrum.

2 In a double shoe brake, the diameter of the brake drum is 350 mm and the contact angle for each shoe is 120°. The coefficient of friction for the brake lining and drum is 0.35. Find the necessary spring force to transmit a torque of 40 N m. The distance of the centre of brake drum from the fulcrum and spring is 250 mm and 300 mm, respectively.

3 A simple band brake is operated by a lever of length 450 mm. The brake drum has a diameter of 600 mm and the brake band embraces 5/8th of the circumference. One end of the band is attached to the fulcrum of the lever while the other end is attached to a pin on the lever 120 mm from the fulcrum. The effort applied to the end of the lever is 2 kN, and the coefficient of friction is 0.30. Find the maximum braking torque on the drum.

4 In the differential band brake, shown in Fig.6.4(b), the diameter of the drum is 900 mm, and the coefficient of friction between the drum and the band is 0.3. The angle of contact is 240°. When a force of 650 N is applied at the free end of the lever, find the maximum and the minimum force in the band and the torque which can be applied by the brake. Take $a = 120$ mm, $b = 100$ mm, and $l = 500$ mm.

5 A vehicle is moving on a level track at a speed of 40 km/h. Its centre of gravity lies at a distance of 0.6 m from the ground level. The wheel base is 2.4 m and the distance of the C.G. from the rear is 1 m. Find the distance travelled by the vehicle before coming to rest when brakes are applied (a) to the rear wheels, (b) to the front wheels, and (c) to all the four wheels. The coefficient of friction between the tyres and the road surface is 0.40.

6 A single plate clutch having both sides effective is required to transmit 45 kW at 1500 rpm. The outer diameter of the plate is limited to 300 mm and the intensity of pressure between the plates is not to exceed 0.07 MPa. Assuming uniform wear and a coefficient of friction 0.35, determine the inside diameter of the plate.

7 A multiplate clutch has three pairs of contact surfaces. The outer and inner radii of the contact surfaces are 150 mm and 80 mm respectively. The maximum axial spring force is limited to 0.9 kN and the coefficient of friction is 0.3. Assuming uniform wear find the power transmitted by the clutch at 1500 rpm.

8 A cone clutch with cone angle 25° is to transmit 8 kW at 750 rpm. The normal intensity of pressure between the contact faces does not exceed 0.15 MPa. The coefficient of friction is 0.25. If face width is $1/5^{th}$ of mean diameter, find (a) the main dimensions of the clutch, and (b) axial force required while running. Take $d_1/d_2 = 2$.

9 A torsion dynamometer is fitted on a turbine shaft to measure the angle of twist. It is observed that the shaft twists 2° in a length of 5 m at 600 rpm. The shaft is solid and has a diameter of 250 mm. If the modulus of rigidity is 84 GPa, find the power transmitted by the turbine.

10 Explain the following:

(a) Working of a railway vacuum brake

(b) Differentiate between absorption and transmission dynamometers and give the constructional and operational details of any one of absorption dynamometer.

11 A single plate clutch is required to transmit 220 kW at 6000 rpm. The clutch facings available provide a coefficient of friction of 0.25 and the average pressure is to be limited to 75 kN/m^2. Determine the dimensions of the working surface of the clutch plate if its maximum dimension is not to exceed 160 mm due to space restrictions.

State clearly the assumptions made and derive the formula used.

12 State the difference between a clutch and a brake as well as a brake and a dynamometer. What are the two main classes of dynamometers? Give one example of each class and explain the operation of one of them. It is required to measure the actual power developed by a hydraulic turbine after it was installed. The turbine was connected to a generator whose rated capacity was 50 MW. How will you measure the output power of the turbine?

13 In a winch the rope supports a load W and is wound round a barrel 450 mm diameter. A differential band brake acts on a drum 800 mm diameter which is keyed to the same shaft as the barrel. The two ends of the band are attached to pins on opposite sides of the fulcrum of the brake lever and at distance of 25 mm and 100 mm from the fulcrum. The angle of lap of the brake band is 250° and coefficient of friction is 0.25. What is the maximum load W which can be supported by the brake when a force of 750 N is applied to the lever at a distance of 3 m from the fulcrum?

14 An automobile is travelling along a curved track of 200 m mean radius. Each of the four road wheels have a mass of 80 kg with a radius of gyration of 0.4 m. The rotating parts of the engine have a mass moment of inertia of 10 kg m^2. The crankshaft rotates in the same direction as the road wheels. The gear ratio of engine to the back wheels is 5:1. The vehicle has a mass of 3000 kg and its centre of gravity is 0.5 m above the road level. The width of the track of the vehicle is 1.5 m. Calculate the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface.

15 A differential band brake, under certain conditions can provide self-locking. Where does this facility find applications?

A differential band brake has a force of 220 N applied at the end of a pedal as shown in Fig.6.21. The coefficient of friction between the band and the drum is 0.4. Angle of lap is 180°.

(a) What is the maximum torque the brake may sustain, for a counter-clockwise rotation, when the force applied at the pedal is 220 N?

(b) If a clockwise torque of 450 N m is applied to the drum, determine the maximum and minimum force in the band.

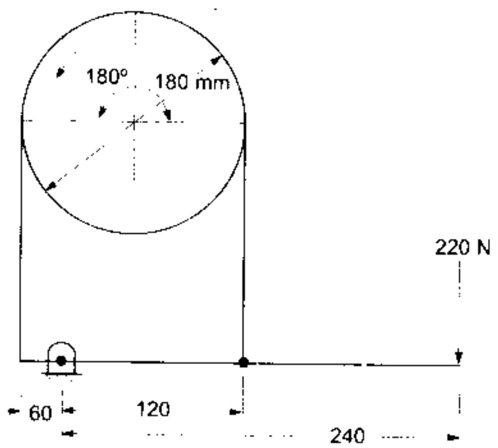


Fig.6.21 Differential band brake mechanism

16 A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 rpm. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm^2. Also determine the axial thrust to be provided by springs. Assume the theory of uniform wear.

17 An automobile single plate clutch consists of a pair of contacting surfaces. The inner and outer diameters of friction plate are 120 mm and 250 mm respectively. The coefficient of friction is 0.25 and the total axial force is 15 kN. Calculate the power transmitting capacity of the plate clutch at 500 rpm using (a) uniform wear theory, and (b) uniform pressure theory.

18 The semi-cone angle of a cone clutch is 12.5° and the contact surfaces have a mean diameter of 80 mm. The coefficient of friction is 0.32.

(a) What is the maximum torque required to produce slipping of the clutch for an axial force of 200 N?

(b) What is the time needed to attain the full speed,

(c) the energy lost during slipping. Motor speed is 900 rpm and the moment of inertia of the flywheel is 0.4 kgm^2.

(d) What are the considerations in the selection of plate clutches and cone clutches?

19 A single plate friction clutch has the following data:

Power developed = 30 kW

Speed = 2400 rpm

Axial load = 1600 N

Outside diameter = 300 mm

Coefficient of friction = 0.32

Overload = 10%

(a) Determine the inside diameter of the clutch plate for its both sides effective and assuming uniform wear.

(b) The moment of inertia of rotating parts attached to driven shaft is 2.5 kg m^2 and average torque is 80% of maximum torque. Determine the time lapse before the engine attains the full speed, if the clutch is suddenly applied.

20 A machine is driven from a constant speed shaft rotating at 270 rpm by a disc friction clutch having both sides effective. The moment of inertia of rotating parts of the machine is 4.5 kg m^2. The diameters of friction plate are 250 mm and 150 mm and the axial pressure applied is 0.075 MPa. Assuming uniform pressure and coefficient of friction to be 0.25, determine the time required for the machine to attain full speed when the clutch is suddenly engaged. Also determine the energy supplied during clutch slip.

21 A crane is required to hold a load of 100 kN. This load is attached to a rope wound round the crane barrel which is 450 mm in diameter. The brake drum which is fixed to the barrel shaft is 600 mm diameter. The band embraces 3/4th of the circumference of the drum and the coefficient of friction between the band and the drum is 0.35. The brake is to be applied by a hand lever above the drum and the operating force acting vertically downwards must not exceed 500 N. Find suitable length of lever on both sides of fulcrum, assuming that one of the bands is attached to the fulcrum pin directly. Figure 6.22 shows the drum.

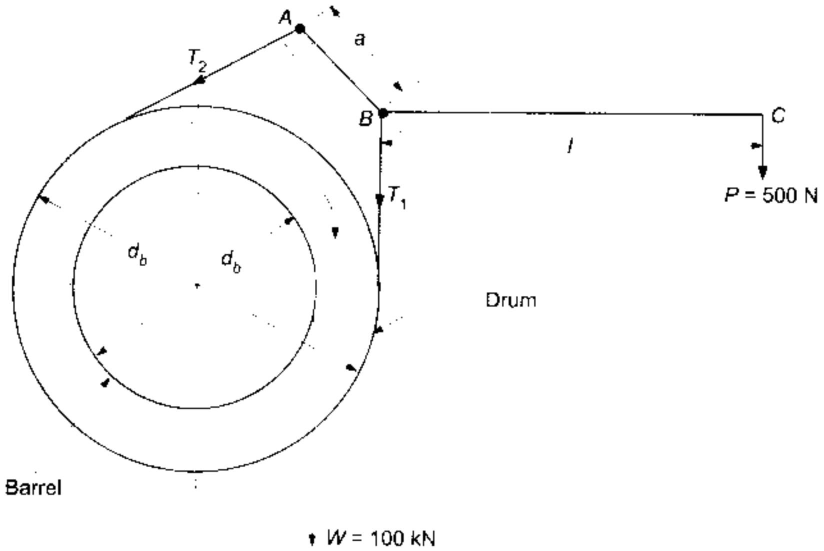
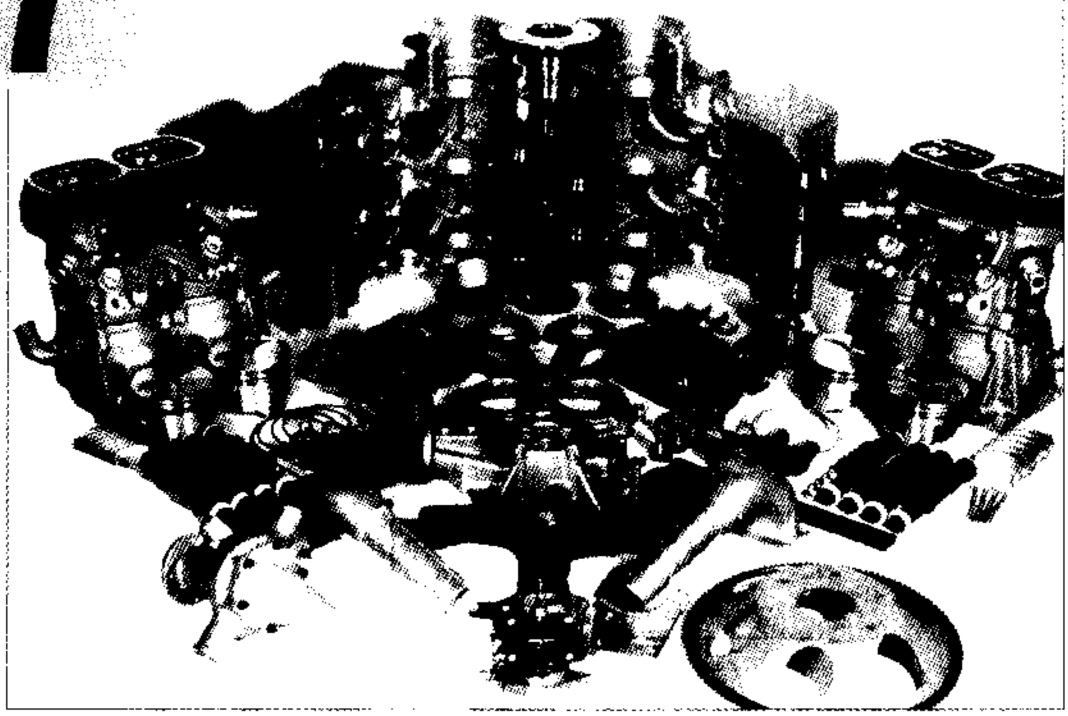


Fig.6.22 Crane

22 The power of a turbine is to be determined by observing that angle of twist of a 6 m long shaft at 520 rpm was 2°. The solid shaft has a diameter of 200 mm and modulus of rigidity 84 GPa. Neglecting the end thrust, determine the power of the turbine.

7



CAMS

7.1 INTRODUCTION

A cam is a rotating or a reciprocating element of a mechanism which imparts rotating, reciprocating or oscillating motion to another element called follower. There is line contact between the cam and the follower, and therefore it forms a higher pair. Cams are used in clocks, printing machines, automatic screw cutting machines, internal combustion engines for operating the valves, shoe making machines etc. In this chapter, we shall study the various types of planar cams from the point of view of drawing their profile and motion analysis.

7.2 TYPES OF CAMS AND FOLLOWERS

The planar cams can be classified as:

Radial or disc cams In these cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis [see Fig.7.1(a)]. These cams can be reciprocating, tangent or circular type.

Cylindrical cams In these cams, the follower reciprocates or oscillates in a direction parallel to the cam axis [see Fig.7.1(b)].

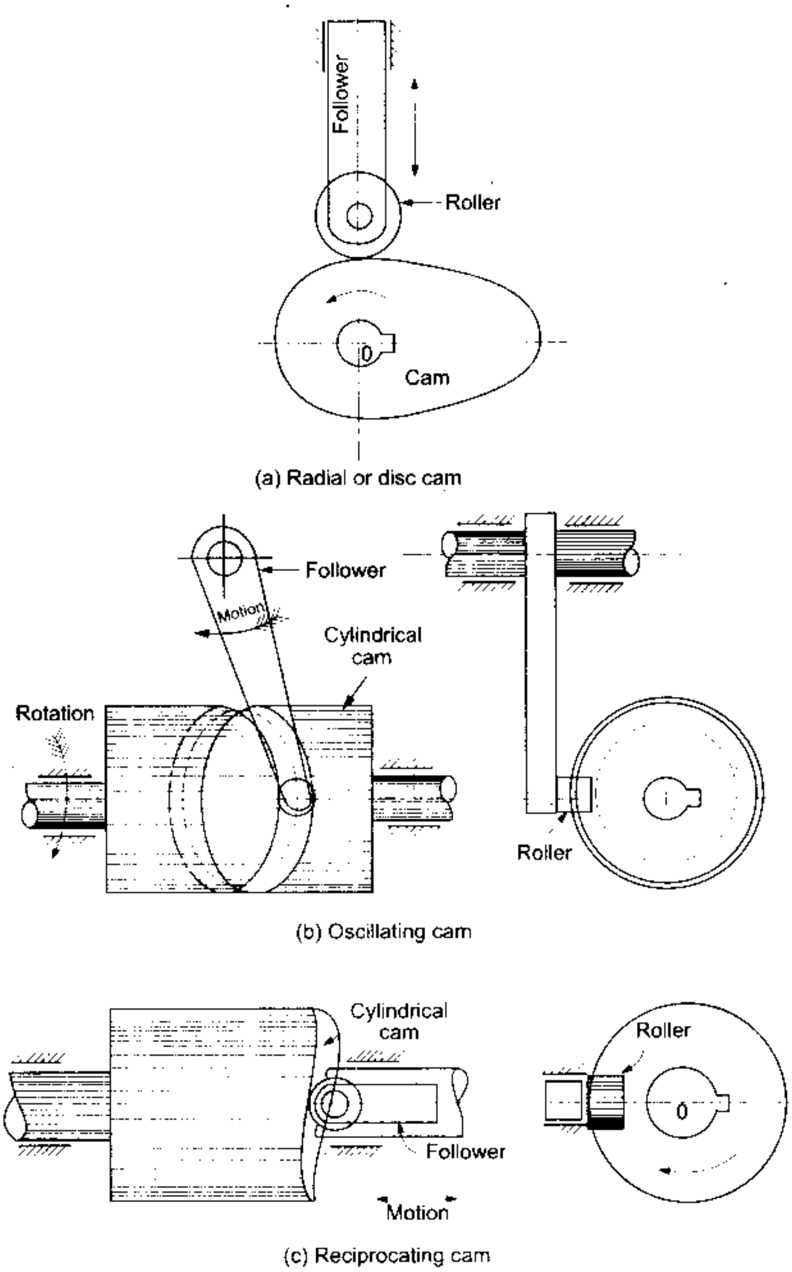
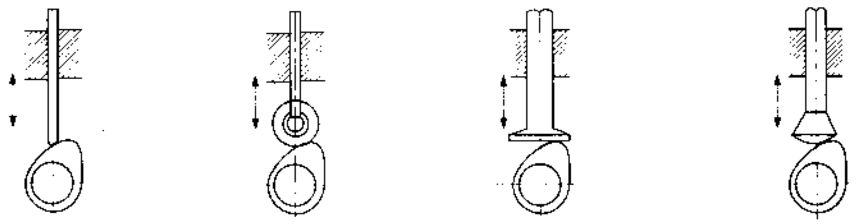


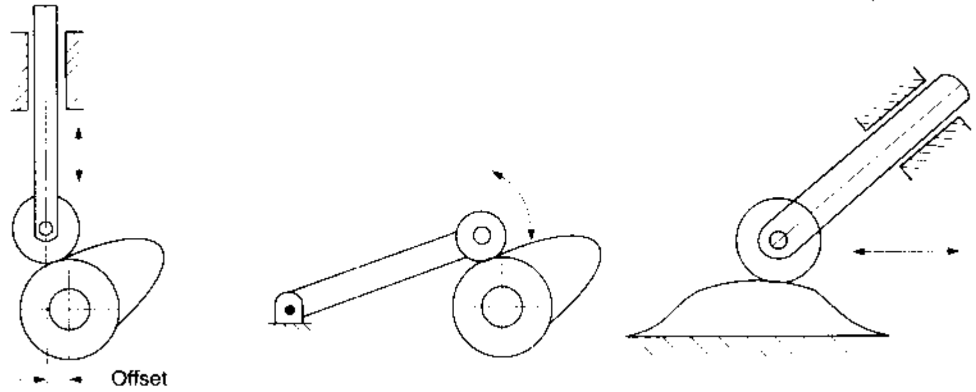
Fig.7.1 Type of cams

Followers can be classified based on:

1. The surface in contact—knife edge, roller, flat-faced (or mush room) and spherical-faced follower [see Fig.7.2(a)–(g)].
2. The motion of the follower—reciprocating or translating, oscillating or rotating follower.
3. The path of motion of follower—radial and off set follower



(a) Knife-edge follower (b) Roller follower (c) Flat-faced (plain/mushroom) follower (d) Spherical-faced follower (mushroom)



(e) Offset follower (f) Oscillating follower (g) Translating follower

Fig.7.2 Types of followers

7.3 CAM NOMENCLATURE

Figure 7.3 shows a radial cam with reciprocating roller follower. The following terms need to be defined in reference to planar cams:

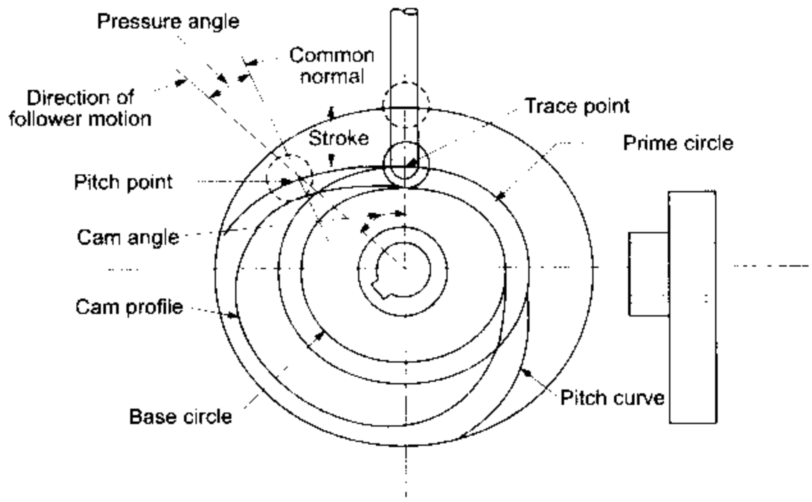


Fig.7.3 Cam nomenclature

Base circle The base circle is the smallest circle that can be drawn to the cam profile from the centre of rotation.

Prime circle The prime circle is the smallest circle drawn to the pitch curve from the centre of rotation of the cam.

Pitch point The pitch point is a point on the pitch curve having the maximum pressure angle.

Pitch circle The pitch circle is the circle drawn through the centre and pitch point.

Trace point The trace point is a reference point on the follower and is used to generate the pitch curve. In the case of a knife edge follower, it is the knife edge and in the case of a roller follower, it is the centre of the roller.

Pitch curve The pitch curve is the curve generated by the trace point as the follower moves relative to the cam.

Cam angle The cam angle is the angle turned through by the cam from the initial position.

Pressure angle The pressure angle is the angle between the direction of the follower motion and a normal to the pitch curve.

Lift Lift is the maximum travel of the follower from the lowest position to the topmost position. It is also called throw or stroke.

7.4 FOLLOWER MOTION

The follower can have following type of motions:

1. Uniform velocity
2. Simple harmonic motion (SHM)
3. Uniform acceleration and deceleration
4. Cycloidal motion

We shall discuss these motions now.

7.4.1 Simple Harmonic Motion

For the SHM shown in Fig.7.4.

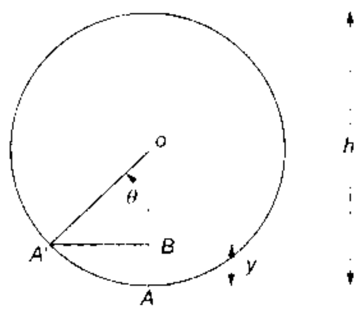


Fig.7.4 SHM of follower

Let h = lift of the follower

θ = angle turned through by the crank from given datum

y = displacement of follower

Then $$y = OA - OB = \frac{h(1-\cos\theta)}{2}$$

Velocity, $$v = \frac{dy}{dt} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{d\theta}{dt}\right)} = \left(\frac{\omega h}{2}\right)\cdot \sin\theta$$

$$v_{max} = \frac{\omega h}{2}$$

where ω is the angular speed of the cam.

Uniform velocity of a particle moving on the circumference of a circle

$$= \frac{\pi h}{2t} = \left(\frac{\pi h}{2}\right)\cdot\left(\frac{\omega}{\theta_1}\right)$$

Maximum velocity of follower during ascent $= \left(\frac{\pi h}{2}\right)\cdot\left(\frac{\omega}{\theta_1}\right)$ (7.1a)

Maximum velocity of follower during descent $= \left(\frac{\pi h}{2}\right)\cdot\left(\frac{\omega}{\theta_3}\right)$ (7.1b)

Acceleration, $$a = \frac{dv}{dt} = \left(\frac{\omega^2 h}{2}\right)\cdot\cos\theta$$

Maximum acceleration, $$a_{max} = \frac{\omega^2 h}{2} = \frac{2v^2}{h}$$

Centripetal acceleration during ascent $= \frac{2v^2}{h}$ (7.2a)

$$= \left(\frac{\pi\omega}{\theta_1}\right)^2\cdot\left(\frac{h}{2}\right) \quad (7.2b)$$

Centripetal acceleration during descent $= \left(\frac{\pi\omega}{\theta_3}\right)^2\cdot\left(\frac{h}{2}\right)$ (7.2c)

The motion of the follower is shown in Fig.7.5.

7.4.2 Motion with Uniform Acceleration and Deceleration

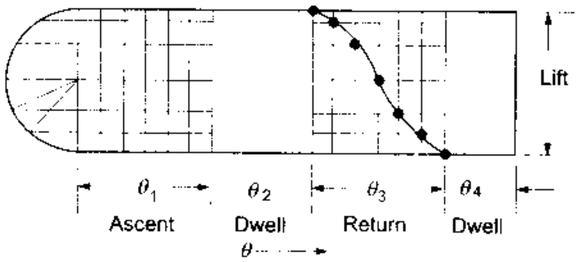
Let a = uniform acceleration or deceleration

Then $$\frac{dv}{dt} = a$$

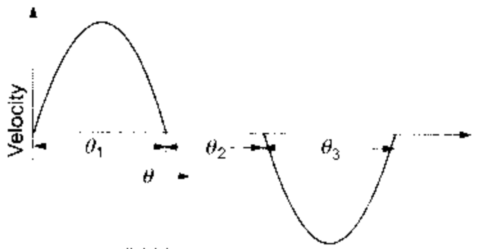
Integrating, we have

$$v = at + C_1$$

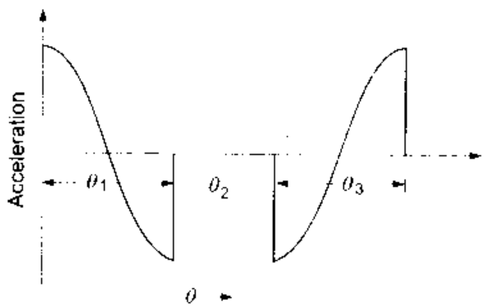
where C_1 is a constant of integration.



(a) Displacement diagram



(b) Velocity diagram



(c) Acceleration diagram

Fig.7.5 Motion of follower moving with SHM

If at $t = 0$, $v = 0$, then $C_1 = 0$. Hence

$$v = at$$

Now
$$v = \frac{dy}{dt} = at$$

Integrating again, we have

$$y = \frac{a\,t^2}{2} + C_2$$

where C_2 is another constant of integration.

As $y = 0$ at $t = 0$, therefore $C_2 = 0$. Hence

$$y = \frac{a\,t^2}{2}$$

Average velocity,
$$v_{avg} = \frac{h}{t}$$

Maximum velocity, $$v_{max} = \frac{2h}{t} = \frac{2h\omega}{\theta_1} \quad \text{during ascent} \tag{7.3a}$$

$$= \frac{2h\omega}{\theta_3} \quad \text{during descent} \tag{7.3b}$$

where θ_1 is the angle of ascent, and θ_3 that of descent.

Maximum acceleration, $$a_{max} = \frac{2v_{max}}{t} = 4\omega^2 \frac{h}{\theta_1^2} \quad \text{during ascent} \tag{7.4a}$$

$$= 4\omega^2 \frac{h}{\theta_3^2} \quad \text{during descent} \tag{7.4b}$$

The motion of the follower is shown in Fig.7.6.

7.4.3 Motion with Uniform Velocity

Let $y = c\theta$

If h = follower rise

β = angle through which the cam is to rotate to rise by h.

Then $h = c\beta$

so that $c = \frac{h}{\beta}$

and $$y = \frac{h\theta}{\beta} \tag{7.5}$$

Velocity, $v = \left(\frac{h}{\beta}\right) \cdot \left(\frac{d\theta}{dt}\right)$

$$= \frac{h\omega}{\beta} \tag{7.6}$$

Acceleration, $a = \left(\frac{h}{\beta}\right) \cdot \left(\frac{d\omega}{dt}\right) = 0$

The displacement, velocity and acceleration are shown in Fig.7.7.

7.4.4 Parabolic Motion

Let for the first half of the motion, y be equal to $c\theta^2$.

For $y = \frac{h}{2}, \quad \theta = \frac{\beta}{2}$

$\therefore$ $\frac{h}{2} = \frac{c\beta^2}{4} \quad \text{or} \quad c = \frac{2h}{\beta^2}$

Hence, $$y = 2h\left(\frac{\theta}{\beta}\right)^2 \tag{7.7}$$

$$v = 4h\frac{\omega\theta}{\beta^2} \tag{7.8}$$

$$a = 4h\left(\frac{\omega}{\beta}\right)^2 = \text{constant} \tag{7.9}$$

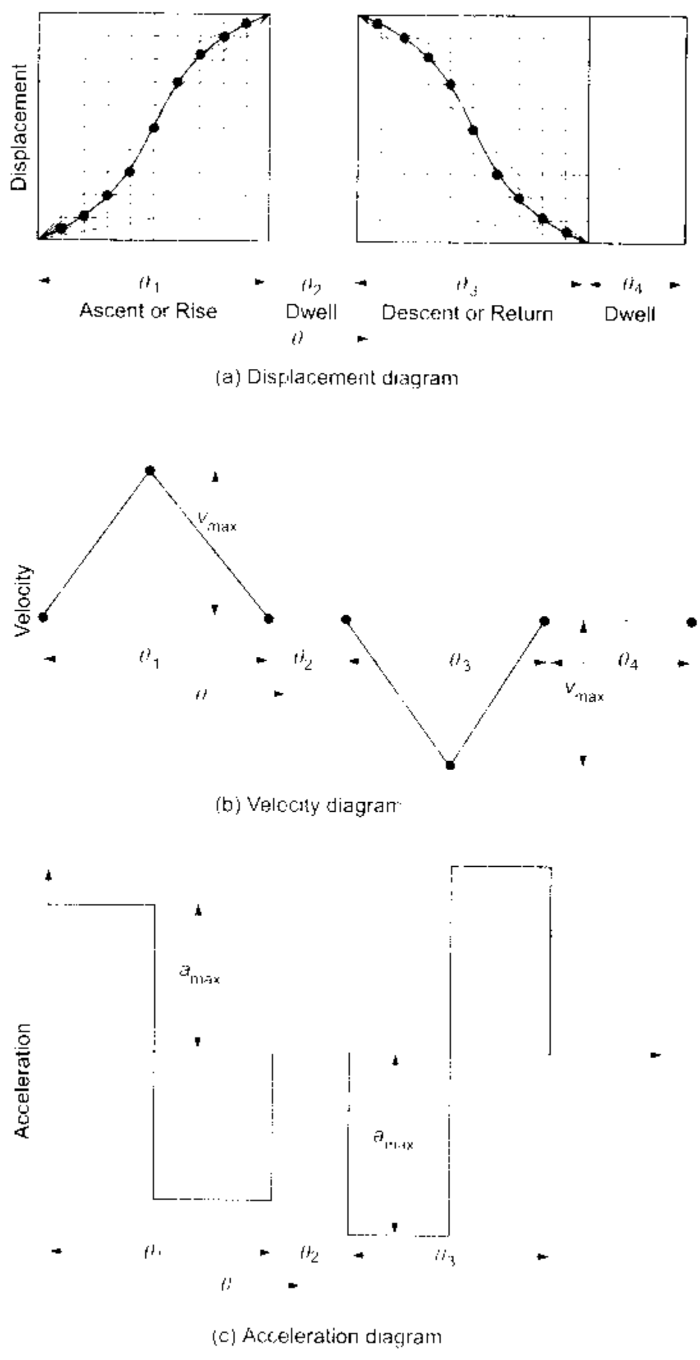


Fig.7.6 Motion of follower moving with uniform acceleration and deceleration

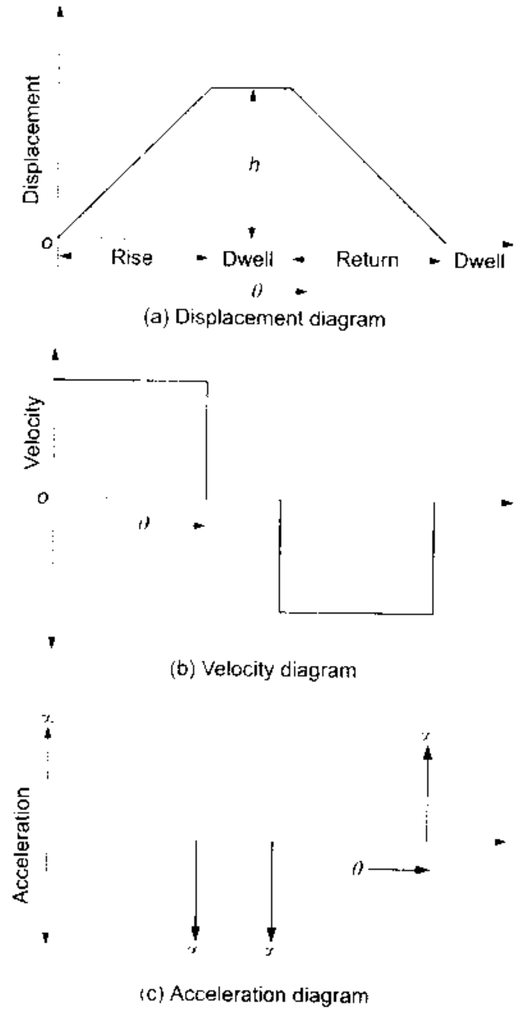


Fig.7.7 Motion of follower moving with uniform speed

For velocity to be maximum, $\theta = \dfrac{\beta}{2}$ and

$$v_{max} = 2h\frac{\omega}{\beta} \qquad (7.10)$$

For the second half, we have

$$y = c_1 + c_2\theta + c_3\theta^2$$

For $\theta = \beta$, $y = h$ and $v = 0$. Also for $\theta = \dfrac{\beta}{2}$, $v = v_{max}$.

Therefore,

$$h = c_1 + c_2\beta + c_3\beta^2$$

$$0 = c_2\omega + 2c_3\omega\beta$$

or

$$0 = c_2 + 2c_3\beta$$

Hence,

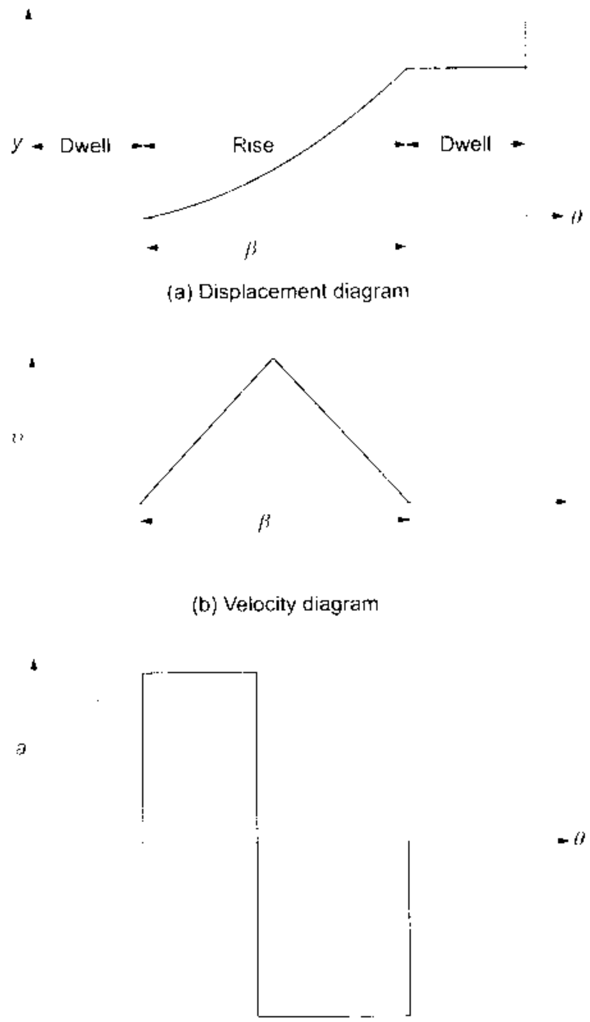
$$c_1 = h,\ c_2 = 4\frac{h}{\beta},\ c_3 = -2\frac{h}{\beta^2}$$

$$y = h\left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right] \tag{7.11}$$

$$v = \left(4h\frac{\omega}{\beta}\right)\left(1 - \frac{\theta}{\beta}\right) \tag{7.12}$$

$$a = -4h\left(\frac{\omega}{\beta}\right)^2 \tag{7.13}$$

The displacement, velocity and acceleration are shown in Fig 7.8.



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig.7.8 Motion of follower moving with parabolic motion

Cycloidal motion

Here
$$y = h\left[\frac{\theta}{\beta} - \left(\frac{1}{2\pi}\right)\sin\left(\frac{2\pi\theta}{\beta}\right)\right] \quad (7.14)$$
$$v = \left(\frac{h\omega}{\beta}\right)\left[1 - \cos\left(\frac{2\pi\theta}{\beta}\right)\right] \quad (7.15)$$
$$a = 2h\pi\left(\frac{\omega}{\beta}\right)^2 \sin\left(\frac{2\pi\theta}{\beta}\right) \quad (7.16)$$

For velocity to be maximum, $a = 0$, or
$$\sin\left(\frac{2\pi\theta}{\beta}\right) = 0$$
or
$$\frac{2\pi\theta}{\beta} = \pi$$
or
$$\theta = \frac{\beta}{2}$$
$$v_{max} = 2h\frac{\omega}{\beta} \quad (7.17)$$

For acceleration to be maximum,
$$\sin\left(2\pi\frac{\theta}{\beta}\right) = 1.$$
or
$$\frac{2\pi\theta}{\beta} = \frac{\pi}{2}$$
or
$$\theta = \frac{\beta}{4}$$
$$a_{max} = 2\pi\omega^2\frac{h}{\beta^2} \quad (7.18)$$

The displacement, velocity and acceleration are shown in Fig.7.9.

To draw the displacement diagram, the following procedure may be adopted:

1. Draw a circle of radius $R = h/2\pi$ with C as centre.
2. Divide the circle into any number of even parts (say six). Project these points horizontally on the vertical centre line of the circle to locate points *a* and *b*.
3. Divide the angular displacement of the cam during outstroke into the same (that is six) number of equal parts as the circle is divided. Draw vertical lines through these points.
4. Join *CBDE*. From *a* and *b* draw lines parallel to *CBDE* to intersect the vertical lines 1, 2 and 4.
5. Join the points of intersection by a smooth curve to get the cycloidal curve during outstroke.

7.5 PROCEDURE FOR DRAWING CAM PROFILE

The following procedure may be adopted to draw the cam profile:

1. Draw the displacement diagram for follower motion.
2. Consider that the cam remains stationary and the follower moves round it in a direction opposite to the direction of cam rotation.

3. Draw the cam base circle and divide its circumference into a number of divisions depending upon the divisions used in the displacement diagram.
4. Draw various positions of follower with dotted lines corresponding to different angular displacements from the radius from which ascent is to commence.
5. Draw a smooth curve tangential to the contact surface in the different positions.

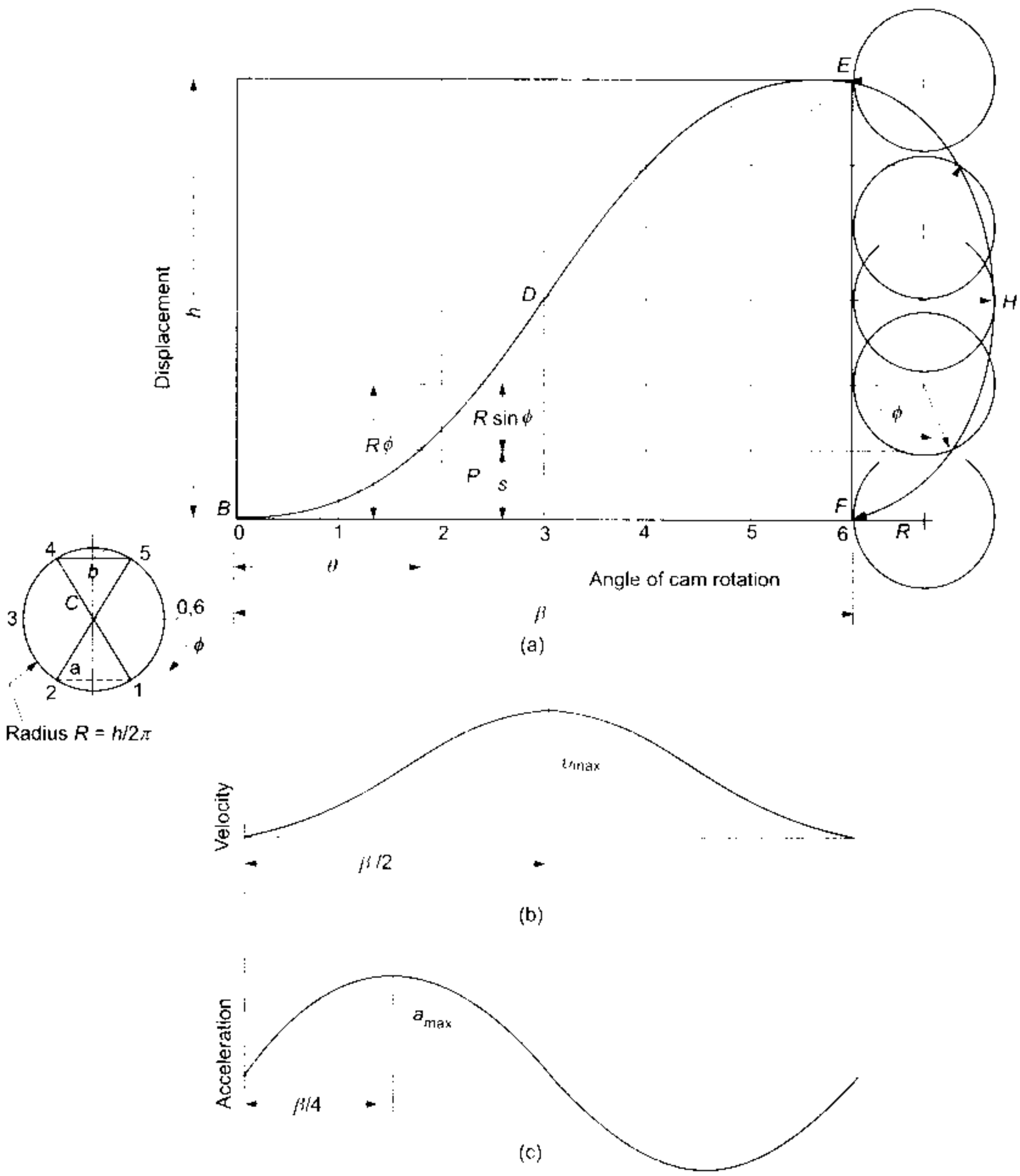


Fig.7.9 Follower moving with cycloidal motion

Example 7.1

Draw the cam profile for the data given below:

Base circle radius of cam	= 50 mm
Lift	= 40 mm
Angle of ascent	= 60°
Angle of dwell	= 40°
Angle of descent	= 90°
Speed of cam	= 300 rpm
Motion of follower	= SHM
Type of follower	= knife-edge

Also calculate the maximum velocity and acceleration during ascent and descent.

Solution

Angular velocity, $\omega = \frac{2\pi \times 300}{60} = 31.416$ rad/s

Maximum velocity, $v_{max} = \left(\frac{\pi h}{2}\right) \cdot \left(\frac{\omega}{\theta_1}\right)$

$$= \left(\frac{\pi \times 40 \times 10^{-3}}{2}\right) \cdot \left[\frac{10\pi}{\left(\frac{\pi}{3}\right)}\right]$$

$= 1.885$ m/s during ascent

$$= \left(\frac{\pi h}{2}\right) \cdot \left(\frac{\omega}{\theta_3}\right)$$

$$= \left(\frac{\pi \times 40 \times 10^{-3}}{2}\right) \cdot \left[\frac{10\pi}{\left(\frac{\pi}{2}\right)}\right]$$

$= 1.257$ m/s during descent

Maximum acceleration, $a_{max} = \left(\frac{\pi\omega}{\theta_1}\right)^2 \cdot \left(\frac{h}{2}\right)$

$$= \left(\frac{\pi \times 30\pi}{\pi}\right)^2 \times 0.02$$

$= 177.65$ m/s^2 during ascent

$$= \left(\frac{\pi\omega}{\theta_3}\right)^2 \cdot \left(\frac{h}{2}\right)$$

$$= \left(\frac{\pi \times 20\pi}{\pi}\right)^2 \times 0.02$$

$= 78.957$ m/s^2 during descent

7.5.1 Displacement Diagram

1. Draw a vertical line 0–6 equal to the lift of 40 mm.
2. Draw a semicircle on this line and divide the semicircle into six equal parts of 60° each.
3. Choosing a scale of 1 mm = 2°, mark the angles of ascent, dwell, descent and dwell as 60°, 40°, 90° and 170° respectively.
4. Divide the angles of ascent and descent into six equal parts and draw vertical lines at those points parallel to 0–6.
5. Draw horizontal lines from points 0 to 6 on the semicircle to intersect the vertical lines.
6. Join the points of intersection with a smooth curve to get the displacement diagram.

The displacement diagram is shown in Fig.7.10(a).

7.5.2 Cam Profile

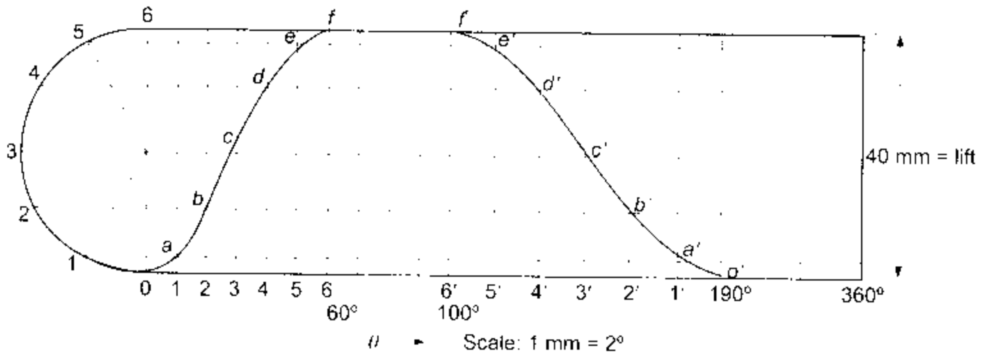
1. Draw a circle of radius equal to the base circle radius of 50 mm.
2. Draw angles of ascent, dwell and descent equal to 60°, 40° and 90° respectively. Divide the angles of ascent and descent into six equal parts and draw radial lines.
3. Mark points 0–6 and 6′–0′ on the base circle in the angles of ascent and descent respectively.
4. Measure distances *1a, 2b, 3c, 4d, 5e* and *6f* from the displacement diagram and cut off corresponding distances on the radial lines. Repeat the same procedure during the descent.
5. Join the points so obtained by a smooth curve to get the cam profile.

The cam profile has been drawn in Fig.7.10(b).

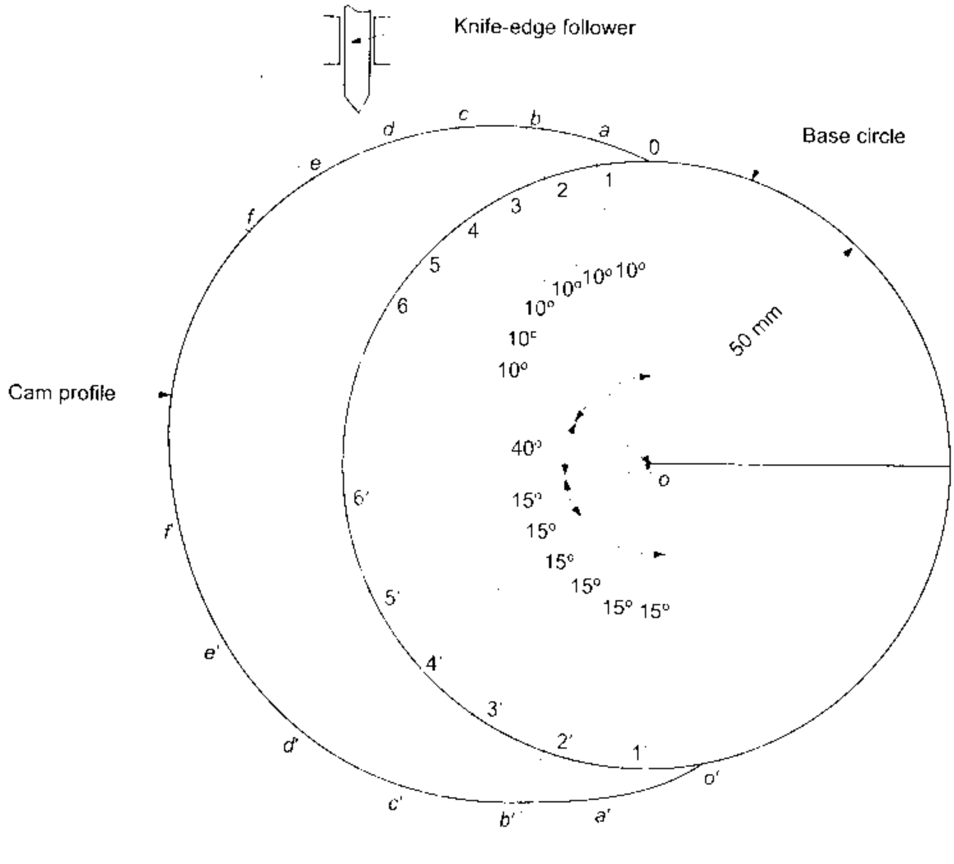
7.6 RADIAL CAM WITH ROLLER FOLLOWER

A radial cam with roller follower has been shown in Fig.7.11. The following steps may be used to draw the cam profile:

1. Draw the base circle.
2. Draw the follower in its 0° position, tangent to the base circle.
3. Draw the reference circle through the centre of the follower in its 0° position.
4. Draw radial lines from the centre of the cam, corresponding to the vertical lines in the displacement diagram.
5. Transfer displacements *1a, 2b, 3c, ...* from the displacement diagram to the appropriate radial lines, measuring from the reference circle.
6. Draw in the follower outline on the various radial lines.
7. Draw a smooth curve tangent to these follower outlines.

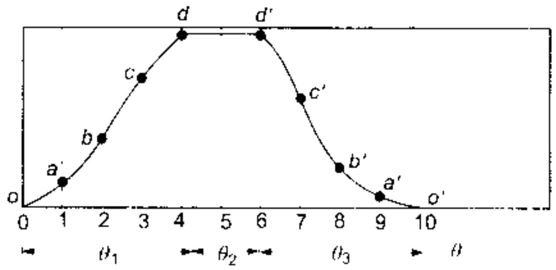


(a) Displacement diagram

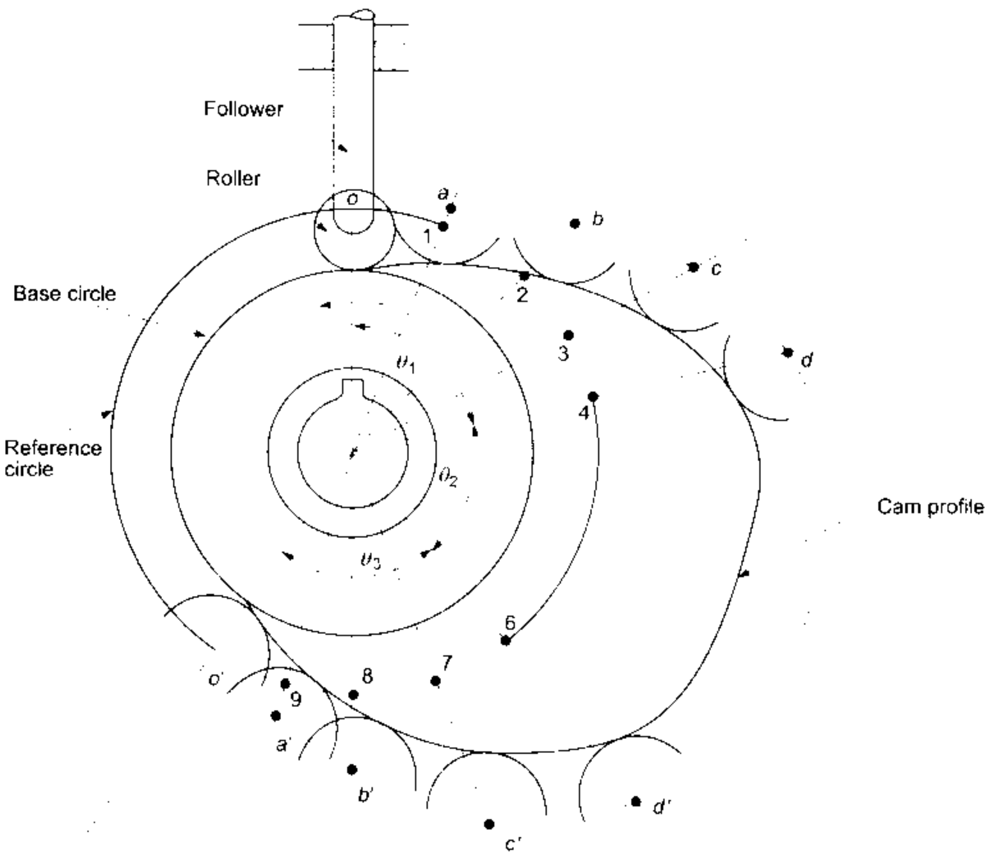


(b) Cam profile

Fig.7.10 Cam with Knife-edge follower having SHM



(a) Displacement diagram



(b) Cam profile

Fig.7.11 Radial cam with roller follower

Example 7.2

A cam of base circle 50 mm is to operate a roller follower of 20 mm diameter. The follower is to have SHM. The angular speed of the cam is 360 rpm. Draw the cam profile for the cam lift of 40 mm. Angle of ascent = 60°, angle of dwell = 40°, and angle of descent = 90°, followed by angle of dwell again. Also calculate the maximum velocity and acceleration during ascent and descent.

■ **Solution**

Angular velocity of cam, $\omega = \dfrac{2\pi \times 360}{60} = 12\pi$ rad/s

Maximum velocity, $v_{max} = \left(\dfrac{\pi h}{2}\right) \cdot \left(\dfrac{\omega}{\theta_1}\right)$

$$= \left(\frac{\pi \times 40 \times 10^{-3}}{2}\right) \cdot \left[\frac{12\pi}{\left(\frac{\pi}{3}\right)}\right]$$

$$= 2.262 \text{ m/s during ascent}$$

$$= \left(\frac{\pi h}{2}\right) \cdot \left(\frac{\omega}{\theta_3}\right)$$

$$= \left(\frac{\pi \times 40 \times 10^{-3}}{2}\right) \cdot \left[\frac{12\pi}{\left(\frac{\pi}{2}\right)}\right]$$

$$= 1.507 \text{ m/s during descent}$$

Maximum acceleration, $a_{max} = \left(\dfrac{\pi\omega}{\theta_1}\right)^2 \cdot \left(\dfrac{h}{2}\right)$

$$= \left(\frac{\pi \times 36\pi}{\pi}\right)^2 \times 0.02$$

$$= 255.82 \text{ m/s}^2 \text{ during ascent}$$

$$= \left(\frac{\pi\omega}{\theta_3}\right)^2 \cdot \left(\frac{h}{2}\right)$$

$$= \left(\frac{\pi \times 24\pi}{\pi}\right)^2 \times 0.02$$

$$= 113.7 \text{ m/s}^2 \text{ during descent}$$

Draw the displacement diagram as explained in Example 7.1.

7.6.1 Cam Profile

1. Draw the base circle with 50 mm radius and another circle with base circle plus roller radius of 60 mm.
2. Draw the angle of ascent of 60°, angle of dwell of 40° and angle of descent of 90°.
3. Divide the angle of ascent and descent into six equal parts and draw radial lines for these angles.
4. Cut off distances on the radial lines as measured from the displacement diagram with roller centre path as the datum.
5. At these points, draw roller circles.
6. Draw a smooth curve tangential to the roller circles to obtain the cam profile.

The cam profile has been drawn in Fig.7.12.

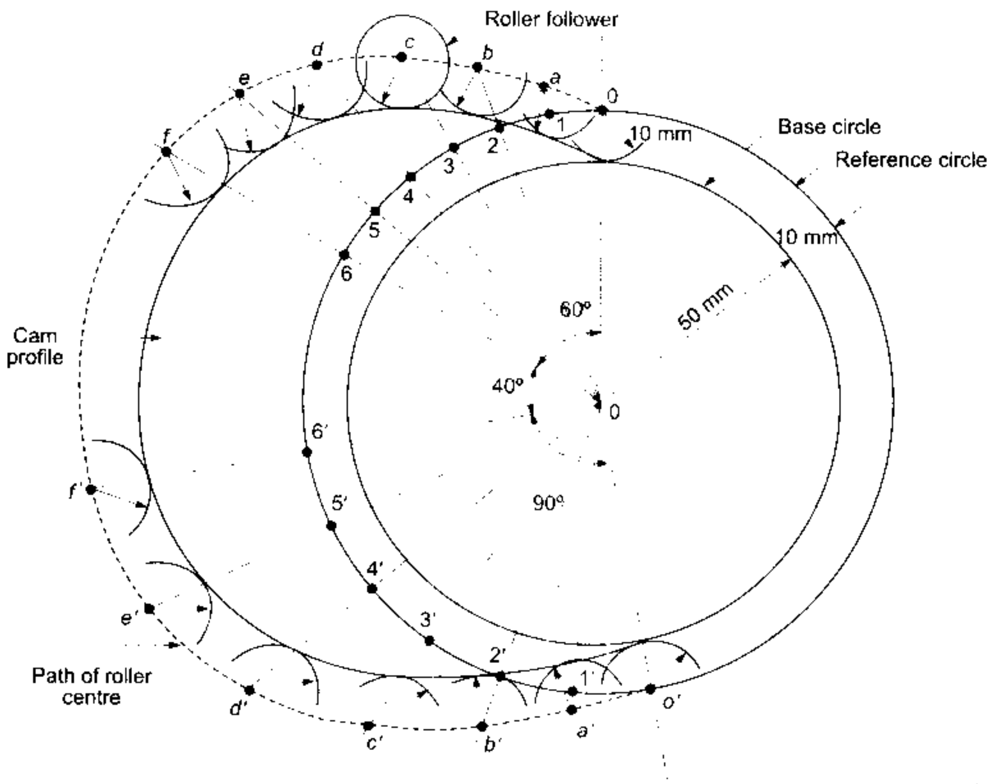
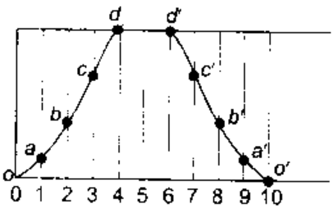


Fig.7.12 Cam with roller follower having SHM

7.7 CAM WITH OFFSET ROLLER FOLLOWER

A cam with offset roller follower is shown in Fig.7.13. The following steps may be used to draw the cam profile:

1. Draw the base circle.
2. Draw the follower in its 0° position, tangent to the base circle.
3. Draw the reference circle through the centre of the follower in its 0° position.
4. Draw the offset circle tangent to the follower centre line.
5. Divide the offset circle into a number of divisions corresponding to the divisions in the displacement diagram.
6. Draw tangents to the offset circle at each number.
7. Lay off various displacements *1a*, *2b*, *3c*, . . . along the appropriate tangent lines, measuring from the reference circle.
8. Draw in the follower outlines on the various tangent lines.
9. Draw a smooth curve to these follower outlines.



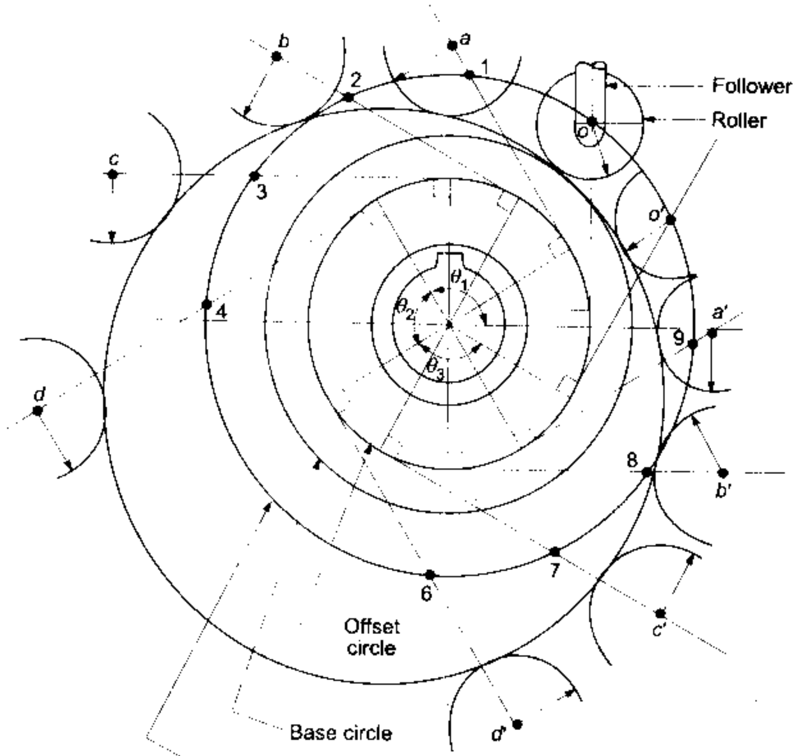


Fig.7.13 Cam with offset roller follower

Example 7.3

A cam operates an offset roller follower. The least radius of the cam is 50 mm, roller diameter is 30 mm, and offset is 20 mm. The cam rotates at 360 rpm. The angle of ascent is 48°, angle of dwell is 42°, and angle of descent is 60°. The motion is to be SHM during ascent and uniform acceleration and deceleration during descent. Draw the cam profile.

Also calculate the maximum velocity and acceleration during descent.

■ **Solution**

$$\omega = \frac{2\pi \times 360}{60} = 12\pi \text{ rad/s}$$

Maximum velocity during descent with uniform acceleration and deceleration,

$$v_{max} = \frac{2h\omega}{\theta_3} \text{ during descent}$$

$$= \frac{2 \times 40 \times 36\pi}{\pi} = 2.88 \text{ m/s}$$

Maximum acceleration,

$$a_{max} = \frac{4\omega^2 h}{\theta_3^2} \text{ during descent}$$

$$= 4 \times (12\pi)^2 \times 40 \times \frac{10^{-3}}{(\frac{\pi}{3})^2} = 207.36 \text{ m/s}^2$$

7.7.1 Cam Profile

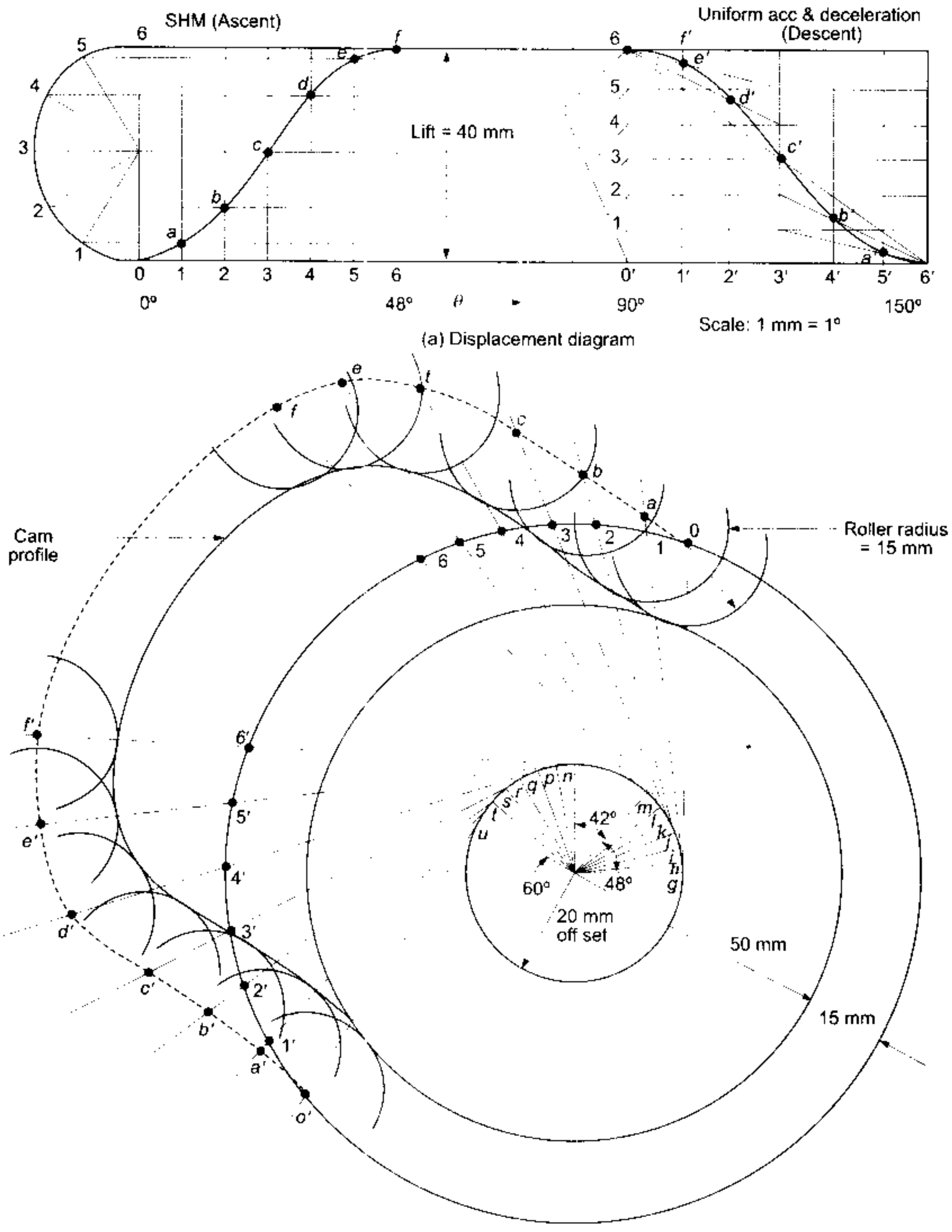
1. Draw the displacement diagram as shown in Fig.7.14(a).
2. Draw the base circle with 50 mm radius. Draw another circle with 65 mm radius to represent the path traced out by the roller centre.
3. Draw a circle 20 mm radius equal to the offset.
4. Divide the angle of ascent and the angle of descent into six equal parts.
5. Draw tangents at the circumference of the offset circle at the above points.
6. From the circle of the path of the roller centre, measure distances from the displacement diagram along the tangential lines.
7. Draw circles at these points equal to the roller radius.
8. Draw a smooth curve tangential to these circles to get the cam profile.

The cam profile has been shown in Fig.7.14(b).

7.8 CAM WITH SWINGING ROLLER FOLLOWER

The cam with swinging roller follower is shown in Fig.7.15. The following steps may be used to draw the cam profile:

1. Draw the base circle.
2. Draw the follower in its 0° position, tangent to the base circle.
3. Draw the reference circle through the centre of the follower.
4. Locate points around the reference circle corresponding to the divisions in the displacement diagram and number them accordingly.
5. Draw a pivot circle through the follower point.
6. Locate the pivot points around the pivot circle corresponding to each point on the reference circle, and number them accordingly.
7. From each of the pivot points, draw an arc where radius is equal to the length of the follower arm.
8. At the zero position, draw the two extreme positions of the follower lever by laying off the arc *AB* equal to the maximum displacement.
9. Lay off the various displacements *1a, 2b, 3c*, ... Along this arc.
10. Rotate each of the points on arc *AB* to its proper position around the cam profile.
11. Draw the follower outline at each of the points just located.
12. Draw a smooth curve tangent to the follower outlines.



(b) Cam profile

Fig.7.14 Cam with offset roller follower having SHM during ascent uniform acceleration and retardation during descent

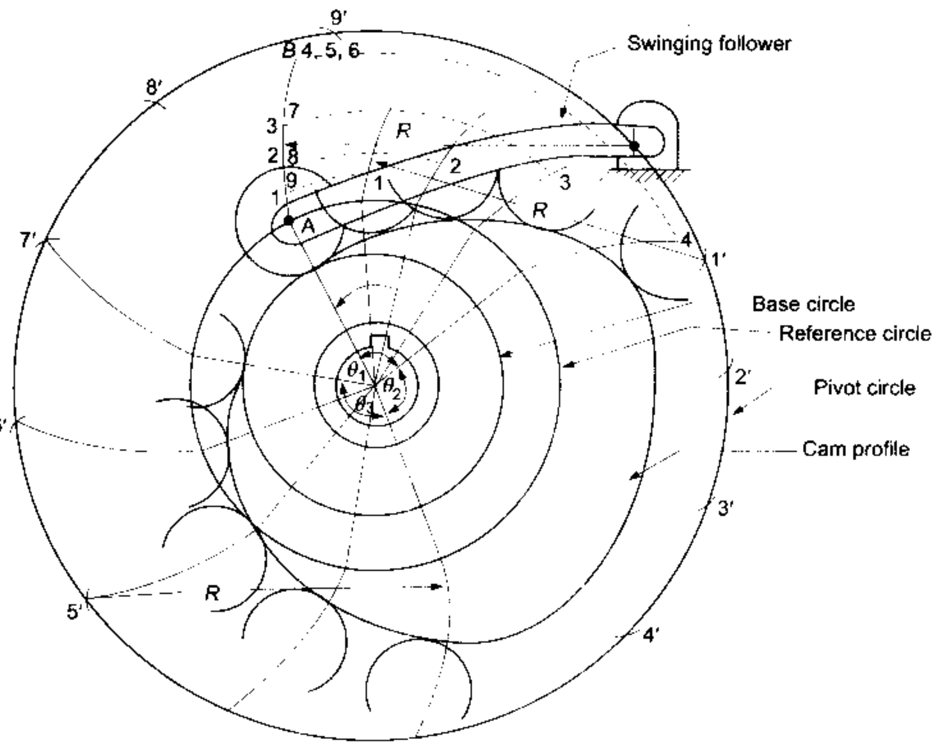


Fig.7.15 Cam with swinging roller follower

Example 7.4

A cam is operating an oscillating roller follower having SHM, as shown in Fig.7.16. Draw the cam profile for the data given below:

Roller centre from cam centre at beginning of ascent	=	60 mm
Angle of ascent	=	60°
Dwell	=	45°
Angle of descent	=	90°
Angle of oscillation of arm during ascent or descent	=	15°

■ Solution

Length of circular arc during movement of follower arm

$$= \frac{80 \times 15\pi}{180} = 20.94 \text{ mm}$$

Length of chord joining the roller centre in extreme position

$$= 2 \times 80 \times \sin 7.5° = 20.88 \text{ mm}$$

For small angle, arc ≈ chord. Therefore, we take displacement = 20.88 mm

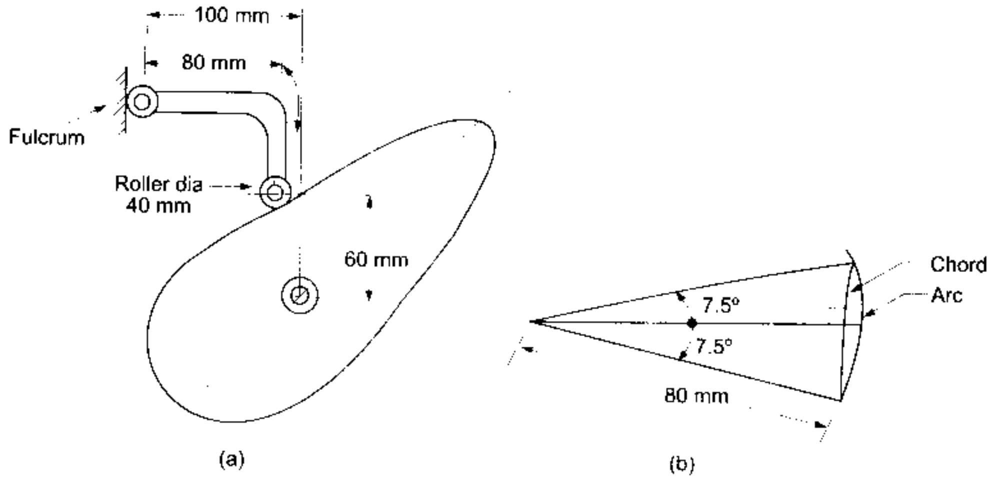


Fig.7.16 Cam with oscillating roller follower

7.8.1 Cam Profile

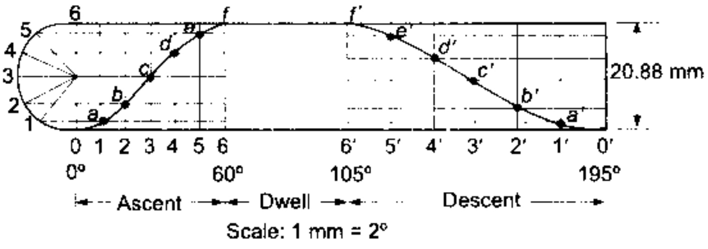
1. Taking lift of cam to be equal to 20.88 mm, draw the displacement diagram, as shown in Fig.7.17(a) and explained in Example 7.1.
2. Draw cam base circle with 40 mm radius, roller centre path with 60 mm radius and another circle with 100 mm radius, which is equal to the distance between cam centre and the fulcrum.
3. Now $OA = 60$ mm and let it be a vertical line. With O and A as centres and radii equal to 100 mm and 80 mm respectively, draw arcs to meet at B. Point B represents the fulcrum position before follower begins to ascend.
4. Mark off $\angle BOC = 60°$, $\angle COD = 45°$, and $\angle DOE = 90°$. Divide the angles of ascent and descent int › six equal parts.
5. With a_1, b_1, c_1, ... As centers, draw arcs of 80 mm radius. From the points of intersection of these arcs with the 60 mm radius circle, mark along the chords of the displacement arcs, distances *pa*, *qb*, *rc*, ... equal to *1a*, *2b*, *3c*, ... of the displacement diagram.
6. Repeat the above procedure for the angle of descent.
7. Join the points *a*, *b*, *c*, ... To obtain the pitch profile of the cam.
8. From the above pitch points, draw the roller radius circles.
9. The envelope of the arcs of roller radius gives the cam profile.

The cam profile has been shown in Fig.7.17(b).

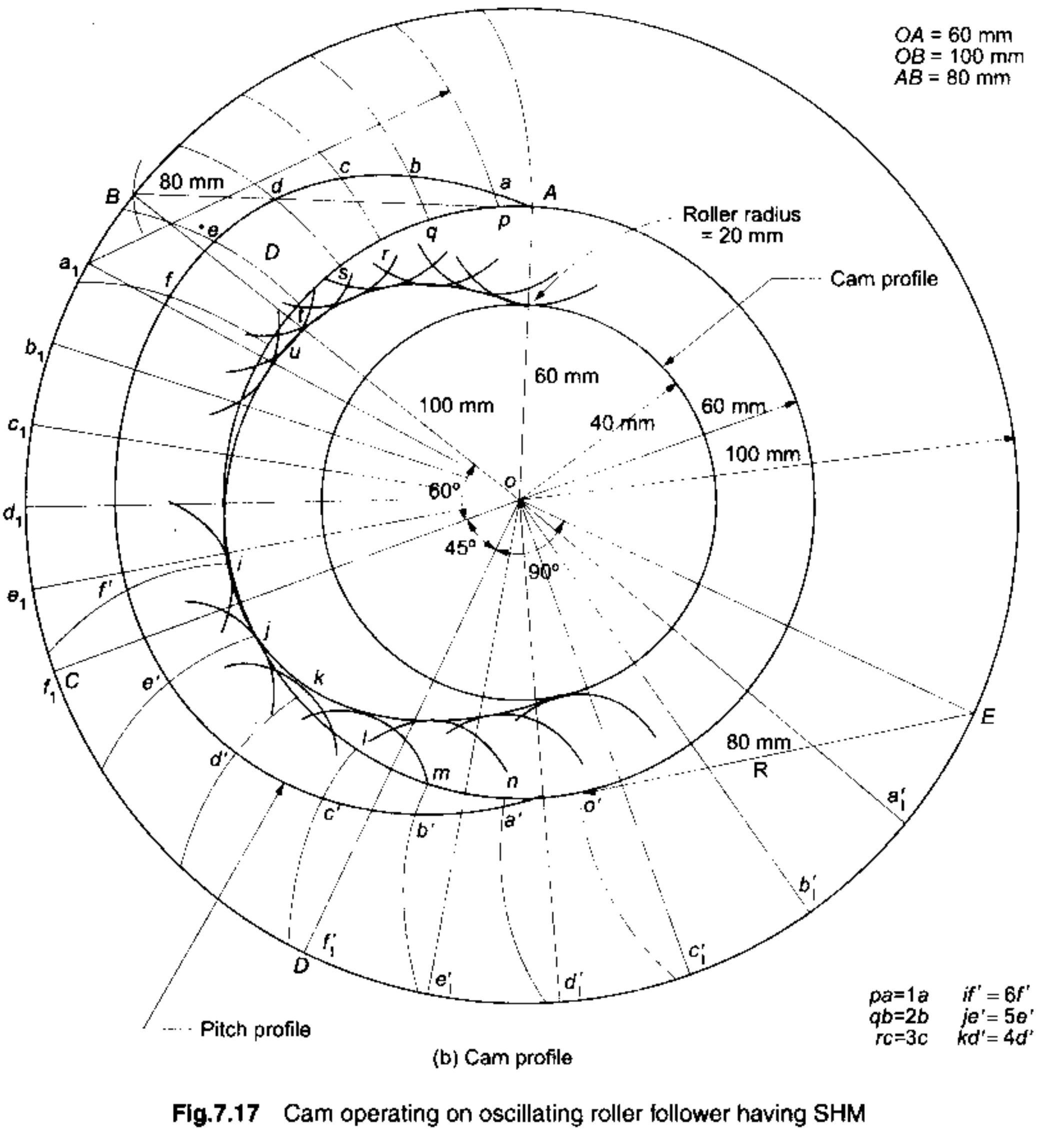
7.9 CAM WITH TRANSLATIONAL FLAT-FACED FOLLOWER

A cam with flat-faced follower is shown in Fig.7.18. The following steps may be followed to draw the cam profile:

1. Draw the base circle, which also serves as the reference circle in this case.
2. Draw the follower in the home position, tangent to the base circle.
3. Draw radial lines corresponding to the divisions in the displacement diagram, and number accordingly.
4. Draw in the follower outline on the various radial lines by laying off the appropriate displacements and drawing lines perpendicular to the radial lines.
5. Draw a smooth curve tangent to the follower lines.

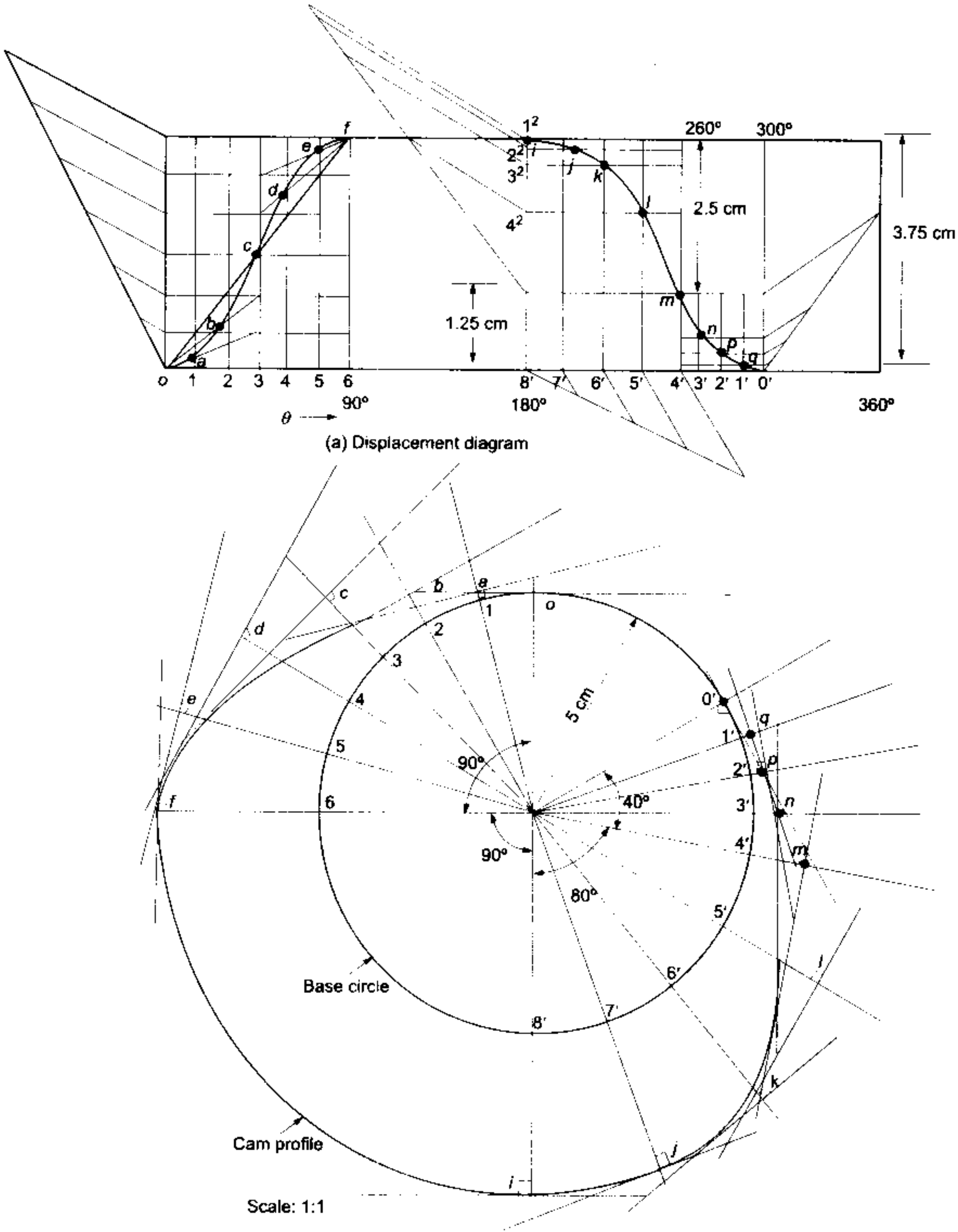


(a) Displacement diagram



(b) Cam profile

Fig.7.17 Cam operating on oscillating roller follower having SHM



(a) Displacement diagram

(b) Cam profile

Fig.7.18

Example 7.5

A cam is to operate a flat-faced follower having uniform acceleration and deceleration during ascent and descent. The least radius of the cam is 50 mm. During descent, the deceleration period is half of the acceleration period. The ascent lift is 37.5 mm. The ascent is for 1/4th period, dwell for 1/4th, descent for 1/3rd, and dwell for the remaining 1/6th period. The cam rotates at 600 rpm. Find the maximum velocity and acceleration during ascent and descent. Draw the cam profile.

Solution

Acceleration period during descent $= \frac{1}{3} \times 360 \times \frac{2}{3} = 80°$

Deceleration period during descent $= 40°$

Distance moved during acceleration period $= \frac{80}{120} \times 37.5 = 25$ mm

Distance moved during deceleration period $= 12.5$ mm

Angular velocity of cam, $\omega = 2\pi \times \frac{600}{60} = 20\pi$ rad/s

Maximum velocity during ascent, $v_{max} = \frac{2\omega h}{\theta_1} = \frac{2 \times 20\pi \times 37.5 \times 10^{-3}}{(\pi/2)}$

$= 3$ m/s

Maximum velocity during descent, $v_{max} = \frac{2h}{t} = \frac{2 \times 37.5 \times 180}{(80 \times \pi)}$

$= 3.375$ m/s

Maximum acceleration during ascent $= 4\omega^2 \frac{h}{\theta_1^2} = \frac{4 \times (20\pi)^2 \times 37.5 \times 10^{-3}}{(\frac{\pi}{2})^2}$

$= 240$ m/s^2

Maximum acceleration during descent $= \frac{v_{max}}{t} = \frac{3.375 \times 360 \times 600}{80 \times 60}$

$= 151.875$ m/s^2

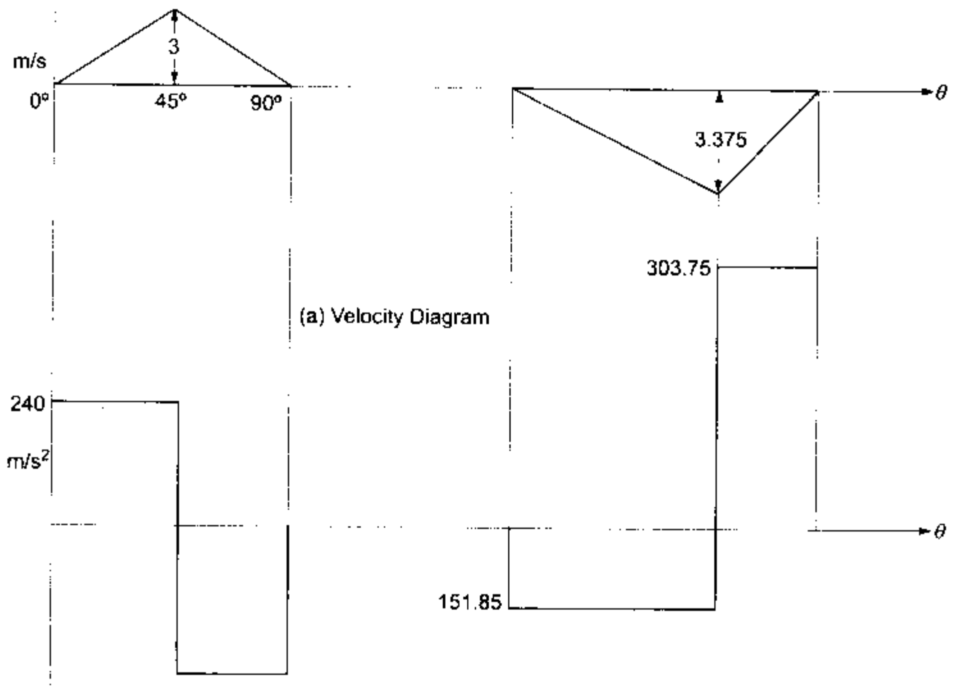
Maximum deceleration during descent $= \frac{v_{max}}{t} = \frac{3.375 \times 360 \times 600}{40 \times 60}$

$= 303.75$ m/s^2

The velocity and acceleration diagrams are shown in Fig.7.19.

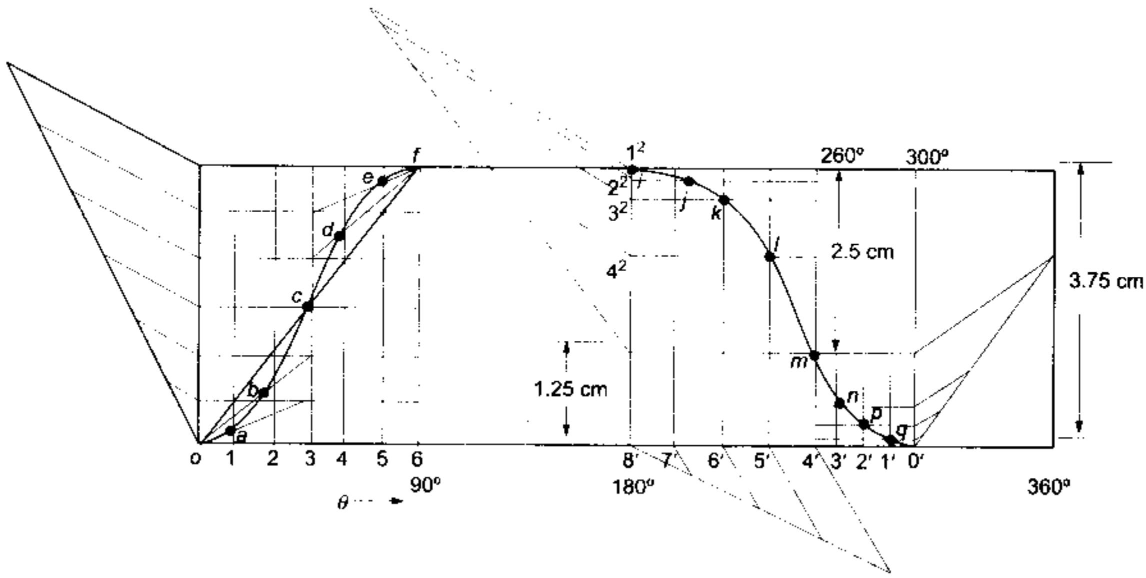
7.9.1 Displacement Diagram

1. Draw a vertical line equal to the lift of 37.5 mm and a horizontal line perpendicular to it, representing the cam angle. Mark angle of ascent = 90°, dwell = 90°, and descent = 120°.
2. Divide the angle of ascent and lift into six equal parts. Join point *O* with three points of intersection of vertical and horizontal lines at point 3, and the other three points with point *f*. Join the points of intersection of these inclined lines with the vertical lines with a smooth curve.
3. Divide the angle of descent into two parts of 80° and 40°. Divide these angles and the lift into four equal parts. Join the intersection of vertical and horizontal lines with a smooth curve. The displacement diagram has been shown in Fig.7.20(a).

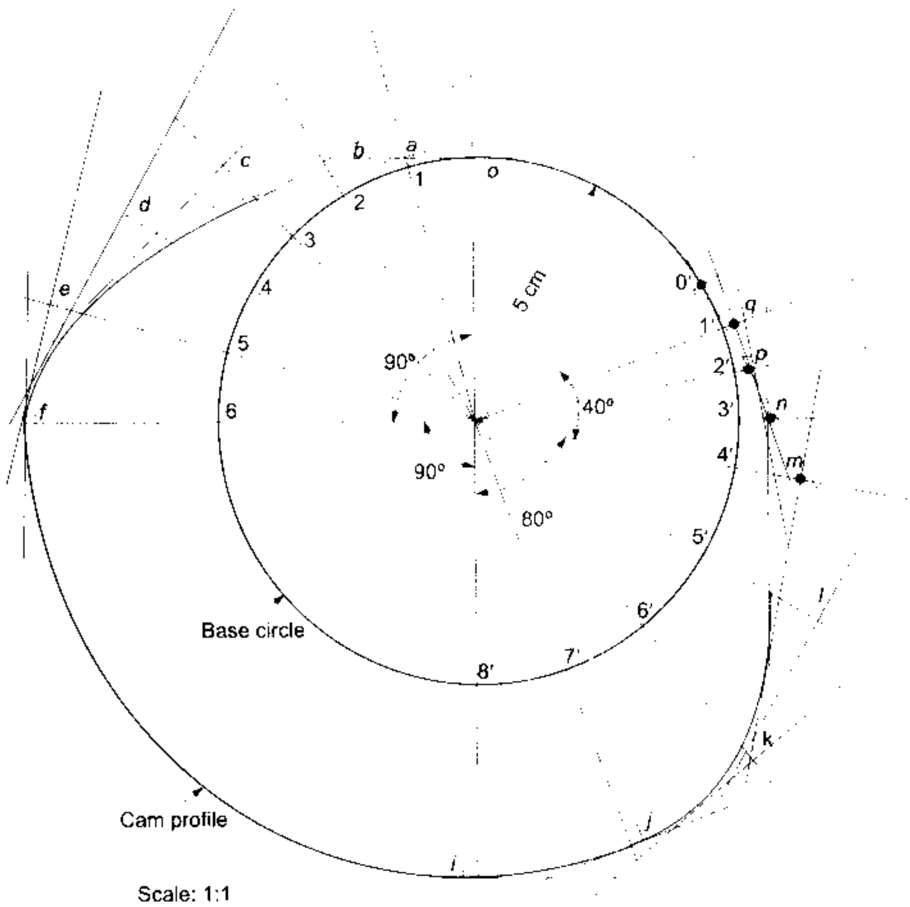


(b) Acceleration diagram

Fig.7.19 Velocity and acceleration diagram for a flat-faced follower having uniform acceleration and deceleration



(a) Displacement diagram



(b) Cam profile

Fig.7.20

7.9.2 Cam Profile

1. Draw the base circle with 50 mm radius.
2. From the vertical line, measure angle of ascent = 90°, dwell = 90° and angle of descent = 120°, in the counter-clockwise direction.
3. Divide the angle of ascent into six equal parts. Divide the angle of descent into 80° and 40°. Divide 80° and 40° angles into four equal parts.
4. From the circumference of the base circle, mark distances *1a*, *2b*, *3c*, . . . along the radial lines at 1, 2, 3, ... Draw perpendiculars at these points to the radial lines. Draw a smooth curve tangential to these perpendiculars to get the cam profile.
5. Repeat this procedure for the descent.

The cam profile has been shown in Fig.7.20(b).

7.10 CAM WITH SWINGING FLAT-FACED FOLLOWER

A cam with swinging flat-faced follower is shown in Fig.7.21. The following steps may be followed to draw the cam profile:

1. Draw the base circle, which in this case also serves the reference circle.
2. Draw the follower in its home position, tangent to the base circle.
3. Draw radial lines corresponding to the divisions in the displacement diagram, and number accordingly.
4. Draw the pivot circle through the follower pivot.
5. Locate the pivot points around the pivot circle.
6. Locate the trace point on the flat face at a radius r from the pivoted point at zero position.
7. At the zero position, draw the two extreme positions of the follower lever laying off the arc *AB* equal to the maximum displacement.
8. Lay off the various displacements, *1a, 2b, 3c*, ... Along this arc.
9. Locate the trace point *a* relative to the cam at the intersection of the arc of radius R, centred at 1 ′ and the arc 1*a* with centre O_2. Establish points *b, c, d*, . . . similarly.
10. The first position of the flat face follower, relative to the cam is the straight line through point *a* that is tangent to the circle of radius r_f, centred at 1 ′. Construct the successive positions of follower face in a similar manner.
11. Draw a smooth curve tangent to the family of straight lines representing the follower face.

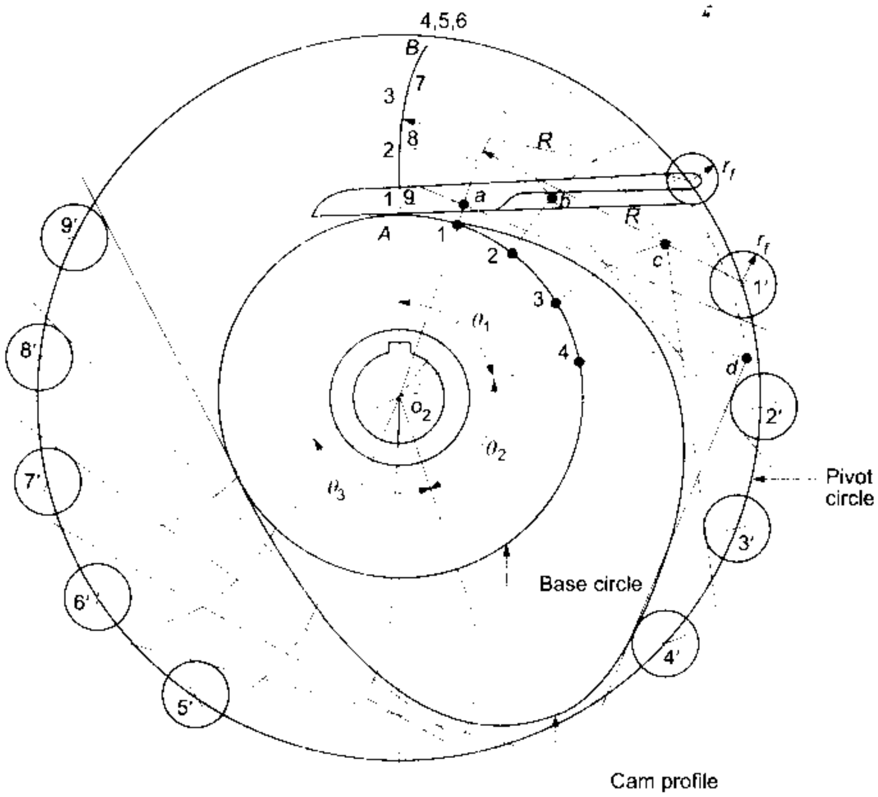


Fig.7.21 Cam with swinging flat-faced follower

Example 7.6

The profile of a cam gives motion to the follower in such a way that it rises through 31.4 mm during 180° of cam rotation with cycloidal motion and returns with cycloidal motion during 180° of cam rotation. Determine the maximum velocity and acceleration of the follower during the outstroke when the cam rotates at 1800 rpm clockwise. The base circle diameter of the cam is 25 mm and roller diameter of the follower is 10 mm. The axis of the follower passes through the cam centre.

■ **Solution**

Stroke, $h = 2\pi R$

Radius of circle generating the cycloid, $R = \frac{31.4}{2\pi} = 5 \text{ mm}$

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

$$v_{max} = \frac{2\omega h}{\beta} = \frac{2 \times 188.5 \times 31.4}{\pi \times 1000} = 3.77 \text{ m/s}$$

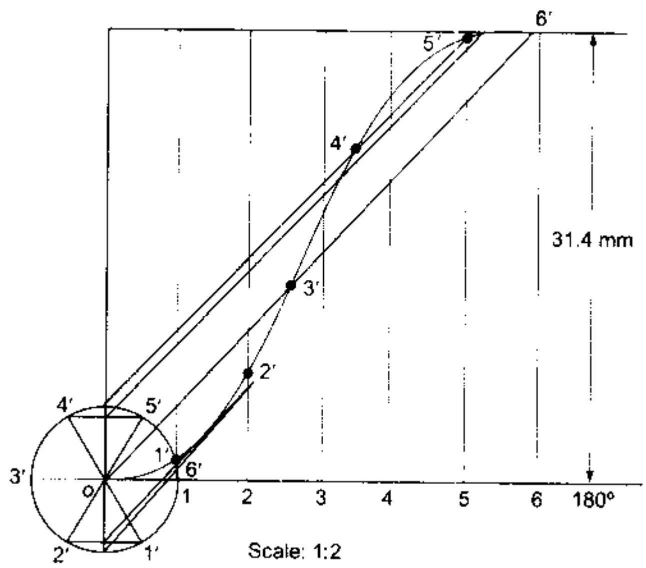
$$a_{max} = \frac{2\pi\omega^2 h}{\beta^2} = \frac{2\pi \times (188.5)^2 \times 31.4}{\pi^2 \times 1000} = 710.3 \text{ m/s}^2$$

The cam profile has been drawn in Fig.7.22.

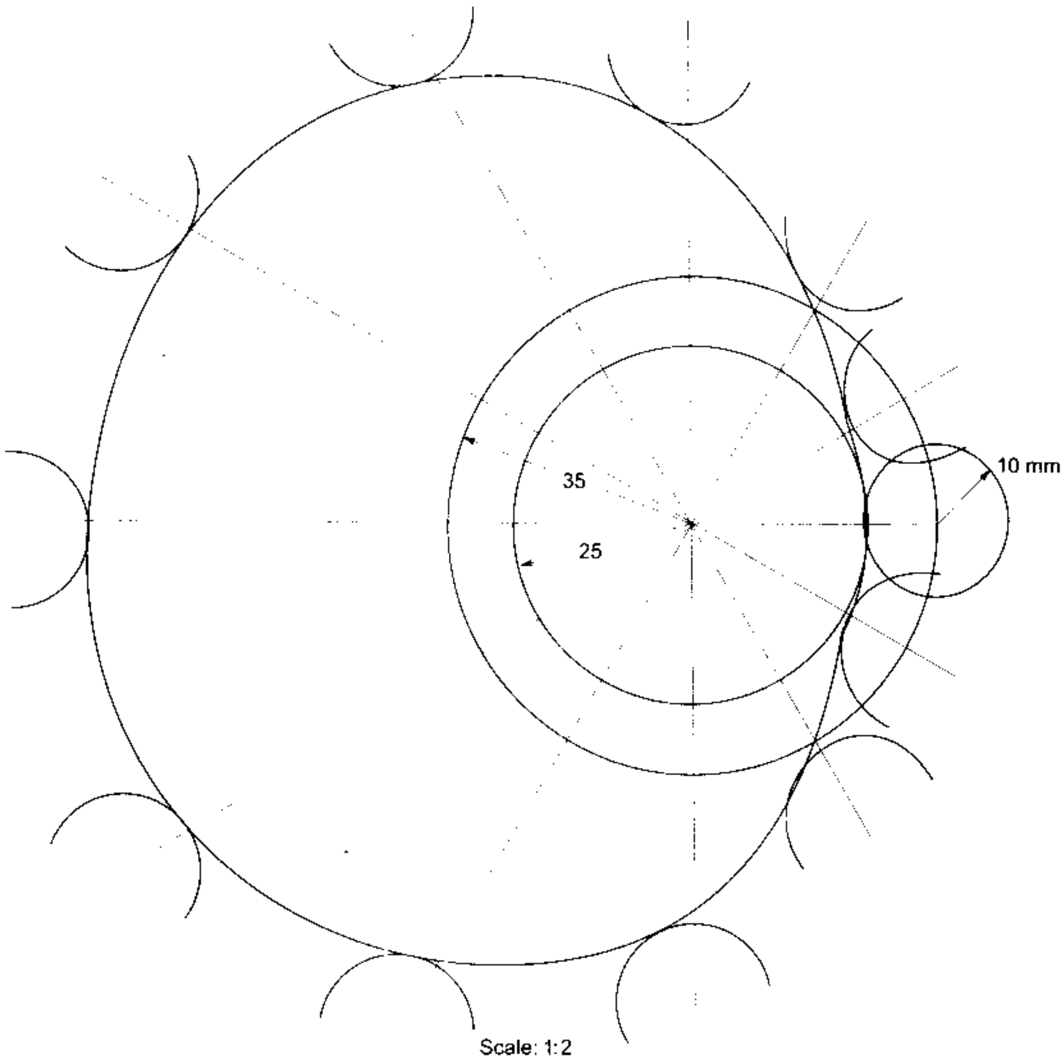
7.11 ANALYTICAL METHODS

7.11.1 Tangent Cam with Roller Follower

A tangent cam having a straight flank and a circular nose with a roller follower is shown in Fig.7.23.



(a) Displacement diagram



Scale: 1:2

(b) Cam profile

Fig.7.22

Let r = distance between cam and nose centres

r_1 = least (base circle) radius of cam

r_2 = nose radius

r_3 = roller radius

$l = r_2 + r_3$

α = angle of ascent

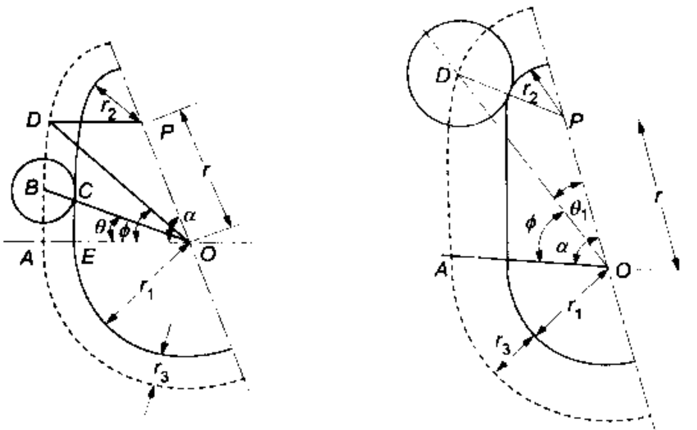
ϕ = angle of contact of cam with straight flank

h = lift of cam

Roller in contact with the straight flank At position B, let θ be the angle turned through by the cam [Fig.7.23(a)]. Then

Lift, $$x = OB - OA = \frac{OA}{\cos\theta} - OA$$
$$= \frac{OA(1-\cos\theta)}{\cos\theta}$$
$$= \frac{(r_1 + r_3)(1-\cos\theta)}{\cos\theta} \quad (7.19)$$

Velocity, $$v = \frac{dx}{dt} = \left(\frac{dx}{d\theta}\right)\cdot\left(\frac{d\theta}{dt}\right)$$
$$= \omega(r_1 + r_3)\left(\frac{-\sin\theta}{\cos^2\theta}\right) \quad (7.20)$$



(a) Contact with straight flank (b) Contact with cricular nose

Fig.7.23

Maximum velocity occurs at $\theta = \phi$.

$$v_{max} = \omega(r_1 + r_3)\left(\frac{-\sin\phi}{\cos^2\phi}\right) \quad (7.21)$$

Acceleration, $$a = \frac{dv}{dt} = \left(\frac{dv}{d\theta}\right)\cdot\left(\frac{d\theta}{dt}\right)$$
$$= \frac{\omega^2(r_1 + r_3)(2-\cos^2\theta)}{\cos^3\theta} \quad (7.22)$$

Minimum acceleration occurs at $\theta = 0°$.

$$a_{min} = \omega^2(r_1 + r_3) \quad (7.23)$$
$$OP + r_2 = r_1 + h$$
$$r = OP = (r_1 + h) - r_2 \quad (7.24)$$
$$\cos\alpha = \frac{r_1 - r_2}{r} \quad (7.25)$$
$$\tan\phi = \frac{r\sin\alpha}{r_1 + r_3} \quad (7.26)$$

Follower in contact with circular nose Referring to Fig.7.23(b),

Let
$$OP = r = \text{constant}$$
$$PD = r_2 + r_3 = l = \text{constant}$$

OPD is a slider–crank chain in which *OP* is the crank, *PD* the connecting rod, and *D* the slider.

Let
$$\theta_1 = \alpha - \theta$$

For a slider–crank chain, the displacement from top dead centre is given by,

$$x = r\left[(1 - \cos\theta) + n - (n^2 - \sin^2\theta)^{0.5}\right]$$

where $n = r/l$.

For the cam mechanism shown in Fig.7.23, we have

$$x = r\left[(1 - \cos\theta_1) + \frac{r_2 + r_3}{r} - \left\{\left(\frac{r_2 + r_3}{r}\right)^2 - \sin^2\theta_1\right\}^{0.5}\right]$$

$$= r\left[(1 - \cos\theta_1) + l - \left(l^2 - r^2\sin^2\theta_1\right)^{0.5}\right] \quad (7.27)$$

$$v = \omega\frac{\mathrm{d}x}{\mathrm{d}\theta} = \omega\left[r\sin\theta_1 + \frac{r^2\sin 2\theta_1}{2}\left(l^2 - r^2\sin^2\theta_1\right)^{0.5}\right] \quad (7.28)$$

$$a = \omega\frac{\mathrm{d}v}{\mathrm{d}t} = \omega^2 r\left[\cos\theta_1 + \frac{\left(l^2 r\cos 2\theta_1 + r^3\sin^4\theta_1\right)}{\left(l^2 - r^2\sin^2\theta_1\right)^{1.5}}\right] \quad (7.29)$$

Example 7.7

The particulars of a symmetrical tangent cam operating a roller follower are given below:

Least radius of cam = 30 mm, roller radius = 20 mm, angle of ascent = 75°, total lift = 20 mm and speed of cam shaft = 600 rpm.

Calculate the (a) principal dimensions of the cam (b) the equation of the displacement curve when the follower is in contact with straight flank and circular nose and (c) the acceleration of the follower at the beginning of lift, where the straight flank merges into the circular nose and at the apex of the circular nose. Assume that there is no dwell between the ascent and return.

■ **Solution**

Here $r_1 = 30$ mm, $r_3 = 20$ mm, $\alpha = 75°$, lift = 20 mm and $N = 600$ rpm.

(a) With reference to Fig.7.23, we have

$$OP + r_2 = 30 + 20$$
$$OP = 50 - r_2$$
$$OQ + r_2 = 30$$
$$OQ = 30 - r_2$$
$$\cos\alpha = \frac{OQ}{OP}$$

$$\cos 75^\circ = \frac{30 - r_2}{50 - r_2}$$

$$0.25882 = \frac{30 - r_2}{50 - r_2}$$

$$12.941 - 0.25882\, r_2 = 30 - r_2$$

Nose radius,
$$r_2 = \frac{17.059}{0.7412} = 23 \text{ mm}$$

Distance between the cam and nose centre,

$$r = OP = 50 - 23 = 27 \text{ mm}$$

$$\tan\phi = \frac{RA}{OA} = \frac{PQ}{OA} = \frac{OP\sin\alpha}{OA} = \frac{23\sin 75^\circ}{50}$$

$$= 0.44432$$

$$\phi = 23.96^\circ$$

(b) Equation of the displacement curve

1. When contact is with the straight flank

$$x = (r_1 + r_3)\left(\frac{1}{\cos\theta} - 1\right) = (30 + 20)\left(\frac{1}{\cos\theta} - 1\right)$$

$$= 50\left(\frac{1}{\cos\theta} - 1\right) \text{ mm}$$

2. When contact is with the circular nose

$$x = r\left[(1 - \cos\theta_1) + l - \left(l^2 - r^2\sin^2\theta_1\right)^{0.5}\right]$$

$$= 23\left[(1 - \cos\theta_1) + (23 + 20) - \left(43^2 - 23^2\sin^2\theta_1\right)^{0.5}\right]$$

$$= 23\left[(1 - \cos\theta_1) + 43 - \left(1849 - 529\sin^2\theta_1\right)^{0.5}\right]$$

where θ_1 is measured from the apex position.

(c) Acceleration of the follower

When in contact with the straight flank,

$$a = \omega^2 (r_1 + r_3)\left[\frac{2 - \cos^2\theta}{\cos^3\theta}\right]$$

$$\omega = \frac{2\pi \times 600}{60} = 62.83 \text{ rad/s}$$

When $\theta = 0^\circ$,

$$a = (62.83)^2(0.030 + 0.020)\left[\frac{2 - 1}{1}\right] = 197.4 \text{ m/s}^2$$

When in contact with the straight flank, $\theta = \phi$,

$$\begin{aligned} a &= \omega^2 (r_1 + r_3) \left[\frac{2 - \cos^2\phi}{\cos^3\phi}\right] \\ &= (62.83)^2(0.03 + 0.02) \left[\frac{2 - \cos^2 23.96°}{\cos^3 23.96°}\right] \\ &= 301.3 \text{ m/s}^2 \end{aligned}$$

When contact is on the circular nose,

$$\begin{aligned} \theta_1 &= \alpha - \phi = 75 - 23.96 = 51.04° \\ a &= \omega^2 r \left[\cos\theta_1 + \frac{l^2 r \cos 2\theta_1 + r^3 \sin^4\theta_1}{(l^2 - r^2\sin^2\theta_1)^{1.5}}\right] \\ &= (62.83)^2 \times 0.023 \left[\cos 51.04° + \frac{0.043^2 \times 0.023 \cos 102.08° + 0.023^3 \sin^4 51.04°}{(0.043^2 - 0.023^2 \sin^2 51.04°)^{1.5}}\right] \\ &= 50.33 \text{ m/s}^2 \end{aligned}$$

When at apex, $\theta_1 = 0°$

$$\begin{aligned} a &= \omega^2 r\left[1 + \frac{r}{l}\right] = (62.83)^2 \times 0.023 \left[1 + \frac{0.023}{0.043}\right] \\ &= 139.36 \text{ m/s}^2 \end{aligned}$$

7.11.2 Graphical Method for a Tangent Cam with Roller Follower

Follower in contact with the straight flank The graphical construction for the tangent cam with roller follower, when the follower is in contact with the straight flank, is shown in Fig.7.24(a).

1. Determination of velocity

   Draw BC parallel to OA. Make $\angle BOC = 90°$. Then $v = \omega \cdot OC$

Proof

In ΔBOC,
$$\frac{OC}{OB} = \tan\theta$$
$$\frac{OA}{OB} = \cos\theta$$

Therefore,
$$OC = \frac{OA \cdot \tan\theta}{\cos\theta} = (r_1 + r_3)\frac{\sin\theta}{\cos^2\theta}$$

Velocity,
$$v = OC \cdot \omega$$

2. Determination of acceleration

   Produce BC so that $BC = CD$ and BO to E. Make $\angle BDE = 90°$. Then, acceleration of follower is OE.

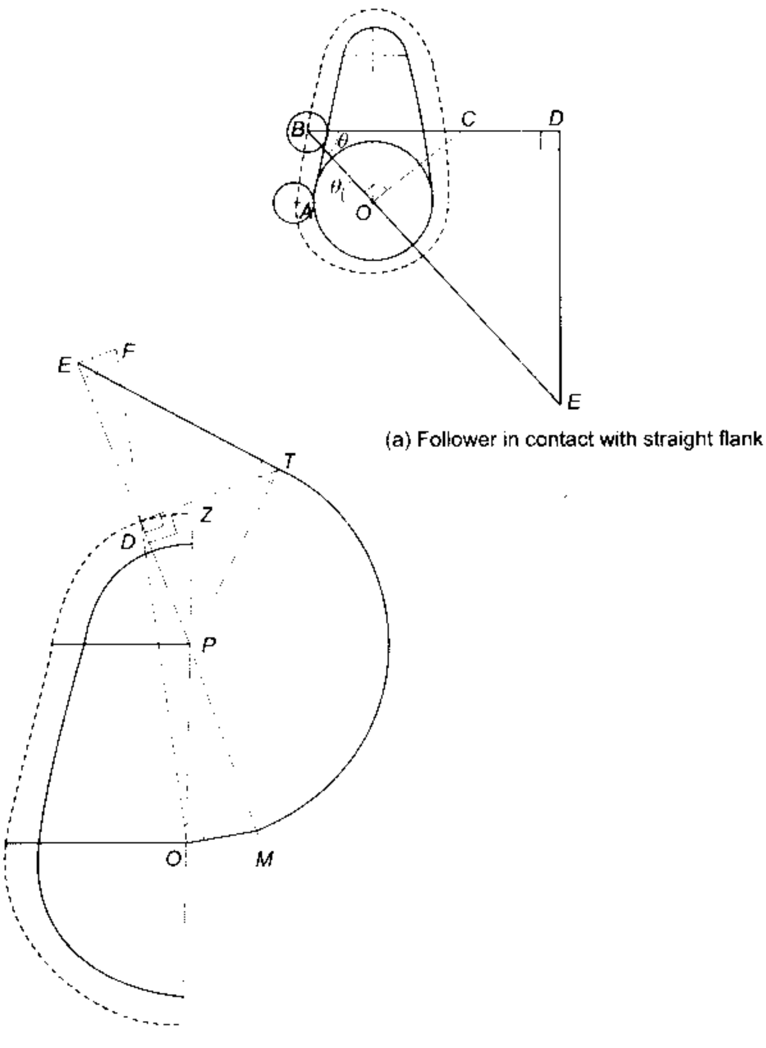


Fig.7.24 Graphical construction for tangent cam with roller follower

Proof

$$\frac{BD}{BE} = \cos\theta$$

Therefore,

$$BE = \frac{BD}{\cos\theta} = \frac{2BC}{\cos\theta}$$

$$= \frac{2BO}{\cos^2\theta} \qquad \left[\because \quad BC = \frac{BO}{\cos\theta}\right]$$

Again

$$OE = BE - OB = \frac{2 \times OB}{\cos^2\theta} - OB$$

$$= OB\left[\frac{2}{\cos^2\theta} - 1\right] = OA\left[\frac{2-\cos^2\theta}{\cos^3\theta}\right]$$

$$= (r_1 + r_3)\left[\frac{2-\cos^2\theta}{\cos^3\theta}\right]$$

$$a = \omega^2 \cdot OE = \omega^2 (r_1 + r_3)\left[\frac{2-\cos^2\theta}{\cos^3\theta}\right]$$

Follower in contact with the circular flank The follower in contact with the circular flank is shown in Fig.7.24(b).

1. Determination of velocity

Draw $DI \perp OD$ meeting OP produced at I. Draw $OM \perp OD$ to meet DP produced at M. Then OM represents velocity of the follower.

Draw $DT \perp DM$. Draw arc MT with centre P and radius PM meeting DT.

Join TP. Draw $ET \perp PT$ and $EF \perp ED$. Then OF represents the acceleration of the follower.

Proof

Triangles DIP and POM are similar since $\angle IPD = \angle OPM$ and

$\angle PID = \angle POM$, because DI is parallel to OM.

$$\frac{v_d}{v_p} = \frac{ID}{IP} = \frac{OM}{OP}$$

or $$v_p = \omega \cdot OP \quad \text{and} \quad v_d = \omega \cdot OM$$

Therefore, $$v_d = \omega \cdot OM$$

2. Determination of acceleration

In $\triangle PTD$, $$\sin\angle PTD = \frac{DP}{TP}$$

In $\triangle PTE$, $$\sin\angle PET = \frac{TP}{PE}$$

Also $$\angle PTD = \angle PET$$

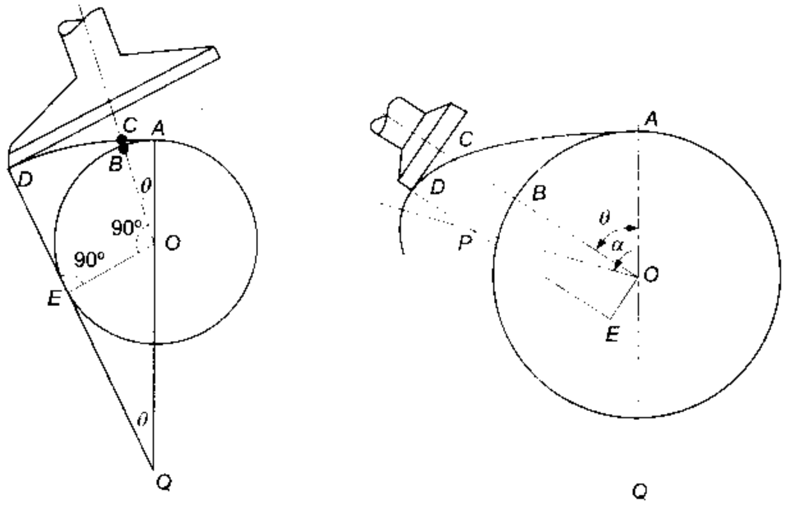
Therefore $$\frac{PD}{TP} = \frac{TP}{PE}$$

or $$PE = \frac{TP^2}{PD} = \frac{PM^2}{PD}$$

As PM represents the velocity of PD, $\frac{PM^2}{PD}$ represents the centripetal acceleration of PD. Then $OPEFO$ represents acceleration image of slider–crank chain OPD, where $\omega^2 \cdot OF$ is the acceleration of roller follower.

7.11.3 Circular Arc Cam Operating Flat-faced Follower

Follower in contact with the circular flank The follower in contact with circular flank is shown in Fig.7.25(a).



(a) Contact with circular flank

(b) Contact with circular nose

Fig.7.25 Circular cam operating flat-faced follower

Let r = distance between cam and nose centers

r_1 = OB = least (base) circle radius of cam

r_2 = nose circle radius

r_3 = QD = flank circle radius

α = angle of ascent

ϕ = angle of contact on circular flank

Displacement of follower,

$$x = OC - OB = DE - r_1$$
$$= (QD - QE) - r_1$$
$$= (r_3 - OQ\cos\theta) - r_1$$
$$= r_3 - (r_3 - r_1)\cos\theta - r_1$$
$$= (r_3 - r_1)(1 - \cos\theta) \quad (7.30)$$

Velocity,

$$v = \frac{dx}{dt} = \left(\frac{dx}{d\theta}\right)\cdot\left(\frac{d\theta}{dt}\right)$$
$$= (r_3 - r_1)\,\omega\sin\theta \quad (7.31)$$

Maximum velocity occurs at $\theta = \phi$.

$$v_{max} = (r_3 - r_1)\,\omega\sin\phi \quad (7.32)$$

Acceleration,

$$a = \frac{dv}{dt} = \left(\frac{dv}{d\theta}\right)\cdot\left(\frac{d\theta}{dt}\right)$$
$$= (r_3 - r_1)\,\omega^2\cos\theta \quad (7.33)$$

Maximum acceleration occurs at $\theta = 0°$.

$$a_{max} = (r_3 - r_1)\,\omega^2 \quad (7.34)$$

Minimum acceleration occurs at $\theta = \phi$.

$$a_{min} = (r_3 - r_1)\omega^2 \cos\phi \tag{7.35}$$

Follower in contact with the circular nose The follower in contact with circular nose is shown in Fig.7.25(b).

$$\begin{aligned} x &= OC - OB = DE - r_1 \\ &= DP + PE - r_1 \\ &= r_2 + OP\cos(\alpha - \theta) - r_1 \\ &= r_2 + r\cos(\alpha - \theta) - r_1 \end{aligned}$$

$$v = \omega \cdot \frac{dx}{d\theta} \tag{7.36}$$

$$= \omega r \sin(\alpha - \theta) \tag{7.37}$$

Velocity is maximum when $\alpha - \theta$ is maximum. This happens when contact changes from the circular flank to the circular nose. Minimum velocity occurs when $\alpha - \theta = 0°$, that is at the apex of the circular nose.

$$v_{min} = 0 \tag{7.38}$$

Acceleration,
$$a = \omega \cdot \frac{dv}{d\theta} = -\omega^2 r \cos(\alpha - \theta) \tag{7.39}$$

Maximum acceleration occurs when $\alpha - \theta = 0°$, that is apex of circular nose.

$$a_{max} = -\omega^2 r \tag{7.40}$$

Minimum acceleration occurs when $\alpha - \theta$ is maximum, i.e. when contact changes from the circular flank to the circular nose.

7.12 CIRCULAR ARC CAM WITH ROLLER FOLLOWER

The circular arc cam with roller follower is shown in Fig.7.26.

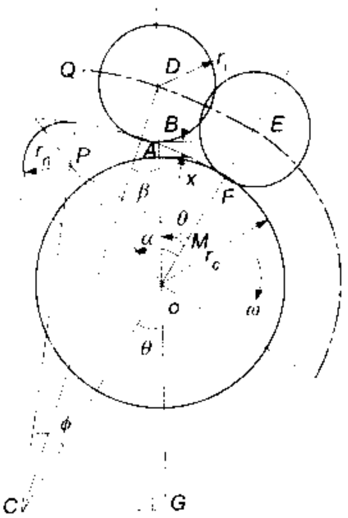


Fig.7.26 Circular arc cam with roller follower on flank

Let $R = CF =$ radius of flank

$r_1 =$ base circle radius of cam

$r_2 =$ nose radius of cam

$r_3 =$ roller follower radius

$h =$ total lift or stroke

$x =$ lift at the instant the cam has rotated by an angle θ

$\alpha =$ semi-angle of action of cam

$\phi =$ angle of action of cam from the beginning of rise to the point it leaves the flank

$\beta = \angle ODC$

7.12.1 Roller Follower on the Flank

Let

$$R - r_1 = A$$

$$R + r_3 = B$$

$$CG = CD \sin\beta = OC \sin\theta$$

or

$$(R + r_3)\sin\beta = (R - r_1)\sin\theta$$

or

$$B \sin\beta = A \sin\theta$$

$$\sin\beta = \left(\frac{A}{B}\right)\sin\theta$$

$$\cos\beta = \left[1 - \left(\frac{A}{B}\right)^2 \sin^2\theta\right]^{0.5}$$

Lift,

$$
\begin{aligned}
x &= AB = OB - OA = OB - OF \\
&= (OD - BD) - OF = (DG - OG) - BD - OF \\
&= (CD\cos\beta - r_3) - r_1 \\
&= (R + r_3)\cos\beta - (R - r_1)\cos\theta - (r_3 + r_1) \\
&= B\cos\beta - A\cos\theta - (B - A) \\
&= \left[B^2 - A^2\sin^2\theta\right]^{0.5} - A\cos\theta - (B - A) \qquad (7.41)
\end{aligned}
$$

Velocity,

$$
\begin{aligned}
v &= \frac{dx}{dt} = \left(\frac{dx}{d\theta}\right)\cdot\left(\frac{d\theta}{dt}\right) \\
&= \omega\left[0.5\left(B^2 - A^2\sin^2\theta\right)^{-0.5}\cdot\left(-2A^2\sin\theta\cos\theta\right) + A\sin\theta\right] \\
&= \omega A\left[\sin\theta - \left\{\frac{(A\sin 2\theta)}{2\left(B^2 - A^2\sin^2\theta\right)^{0.5}}\right\}\right] \qquad (7.42)
\end{aligned}
$$

Acceleration,

$$
\begin{aligned}
a &= \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} \\
&= \omega^2 A\left[\frac{\cos\theta - A\cos 2\theta}{\left(B^2 - A^2\sin^2\theta\right)^{0.5}} - \frac{A^3\sin^2 2\theta}{\left\{4\left(B^2 - A^2\sin^2\theta\right)^{1.5}\right\}}\right] \qquad (7.43)
\end{aligned}
$$

7.12.2 Roller Follower on the Nose

The roller follower on circular nose has been shown in Fig.7.27.

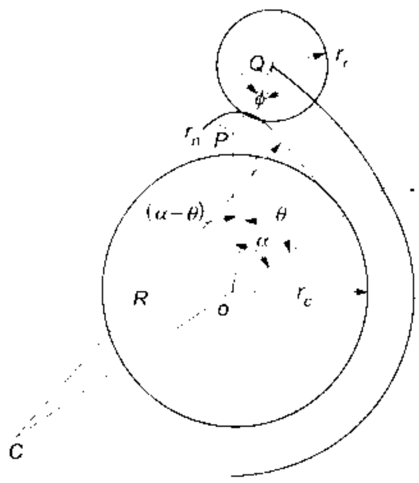


Fig.7.27 Roller follower on circular nose

Let

$$PQ = r_2 + r_3 = l,\ OP = r$$

$$\angle PQO = \beta,\ \angle POQ = (\alpha - \theta) = \theta_1$$

$$v = \omega\left[l\sin\theta_1 + \frac{l^2\sin 2\theta_1}{2\left(l^2 - r^2\sin^2\theta_1\right)}\right] \tag{7.44}$$

$$a = \omega^2\left[-l\cos\theta_1 - \frac{l^2\cos 2\theta_1}{\left(l^2 - r^2\sin^2\theta_1\right)^{0.5}} - \frac{l^4\sin^2 2\theta_1}{4\left(l^2 - r^2\sin^2\theta_1\right)^{1.5}}\right] \tag{7.45}$$

Example 7.8

A cam of circular arc type operates a flat-faced follower of a four stroke engine. The exhaust valve opens 50° before top dead centre and closes 15° after bottom dead centre. The valve lift is 10 mm, base circle radius of cam 20 mm, and nose radius 3 mm. Calculate the maximum velocity, acceleration and retardation, if the cam shaft rotates at 1800 rpm. Also calculate the minimum force required that must be exerted by the spring in order to overcome the inertia of moving parts of mass 0.25 kg.

■ **Solution**

$$2\alpha = \frac{50° + 180° + 15°}{2} = 122.5°$$

$$\alpha = 61.25°$$

$$\omega = 2\pi \times \frac{1800}{60} = 188.5 \text{ rad/s}$$

In Fig.7.28, we have

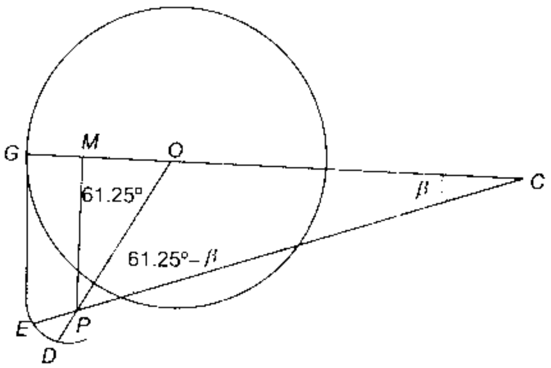


Fig.7.28 Cam of circular ac type with falt-faced follower

$$CP = CE - EP = (OC + OG) - EP = OC + 20 - 3 = OC + 17$$

$$OP = OD - PD = OG + R - r_2 = r_1 + h - r_2 = 20 + 10 - 3 = 27 \text{ mm}$$

$$CP^2 = OC^2 + OP^2 - 2OC \times OP\cos 118.75°$$

$$(OC + 17)^2 = OC^2 + 27^2 - 2 \times OC \times 27 \times \cos 118.75°$$

$$OC = 55 \text{ mm}$$

$$\frac{OC}{\sin 61.25°} = \frac{OP}{\sin\beta} = \frac{CP}{\sin 118.75°}$$

$$\frac{27}{\sin\beta} = \frac{OC + 17}{\sin 118.75°}$$

$$\sin\beta = 27 \times \frac{\sin 118.75°}{72} = 0.32877$$

$$\beta = 19.19°$$

$$R = OC + r_1 = 55 + 20 = 75 \text{ mm}$$

$$v_{max} = \omega(R - r_1)\sin\beta = 188.5 \times 0.055 \times \sin 19.19° = 3.4 \text{ m/s}$$

$$a_{max} = \omega^2(R - r_1) = (188.5)^2 \times 0.055 = 1954.3 \text{ m/s}^2$$

Maximum retardation $= \omega^2 \times OP = (188.5)^2 \times 0.027 = 959.4 \text{ m/s}^2$

Force, $F = 0.25 \times 959.4 = 239.84$ N

Example 7.9

A flat-faced mushroom follower is operated by a symmetrical cam with circular arc flank and nose profile. The axis of tappet passed through the cam axis. Total angle of action is 162°, lift 10 mm and base circle diameter 40 mm. Period of acceleration is half the period of retardation during the lift. The cam rotates at 1200 rpm. Determine (a) the nose and flank radii and (b) the maximum acceleration and retardation during lift.

■ **Solution**

$$\alpha = \frac{162}{2} = 81°, h = 10 \text{ mm}, r_1 = \frac{40}{2} = 20 \text{ mm}, N = 1200 \text{ rpm}$$

Acceleration takes place on the flank and retardation on the nose.

Angle of action on the flank, $\phi = \frac{81}{3} = 27°$

Angle of action on the nose, $\alpha = 2 \times \frac{81}{3} = 54°$

$$\omega = 2\pi \times \frac{1200}{60} = 125.66 \text{ rad/s}$$

In $\triangle COP$ (Fig.7.29), we have

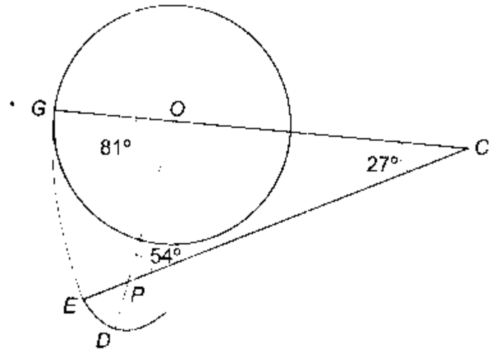


Fig.7.29 Flat-faced mushroom follower

$$CP = CE - EP = OC + OG - EP = OC + 20 - r_2$$
$$OP = OD - PD = OG + h - r_2 = 20 + 10 - r_2 = 30 - r_2$$
$$\frac{OC}{\sin 54°} = \frac{OP}{\sin 27°} = \frac{CP}{\sin 99°}$$
$$\frac{OC}{0.809} = \frac{30 - r_2}{0.454} = \frac{OC + 20 - r_2}{0.9877}$$
$$OC = 0.819(OC + 20 - r_2)$$
$$OC = 90.5 - 4.525\, r_2$$

Also $\quad 30 - r_2 = 0.45965\,(OC - 20 - r_2) \quad (1)$

or $\quad OC = 45.267 - 1.1756\, r_2 \quad (2)$

From (1) and (2), we get

$$r_2 = 13.36 \text{ mm}$$
$$OC = 29.57 \text{ mm}$$
$$OP = 30 - 13.36 = 16.64 \text{ mm}$$

Maximum acceleration $= \omega^2 OC = (125.66)^2 \times 0.02957 = 466.95 \text{ m/s}^2$

Maximum retardation $= \omega^2 OP = (125.66)^2 \times 0.01664 = 262.75 \text{ m/s}^2$

Example 7.10

For a flat-faced follower $\alpha = 75°$, $\phi = 40°$ and the cam is of circular arc type with $r_1 = 30$ mm, $r_2 = 10$ mm, and $r = 40$ mm, $N = 600$ rpm. Combined mass of follower and valve is 5 kg. Calculate the spring force required to close the valve.

■ **Solution**

$$\omega = 2\pi \times \frac{600}{60} = 62.83 \text{ rad/s}$$

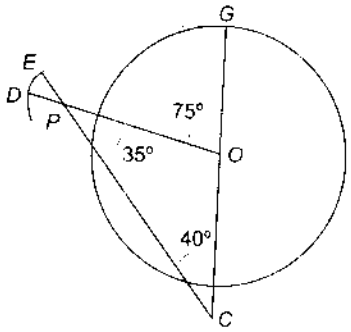


Fig.7.30 Flat-faced foloower and cam

In ΔCOP (Fig.7.30), we have

$$CP = OC + OG - r_2 = OC + 30 - 10 = OC + 20$$
$$OP = r = 40 \text{ mm}$$
$$CP^2 = OC^2 + OP^2 - 2OC \cdot OP\cos(180° - \alpha)$$
$$(OC + 20)^2 = OC^2 + 40^2 - 2 \times OC \times 40 \times \cos 105°$$
$$OC = 62.17 \text{ mm}$$

At the start of the nose, acceleration, $a_{sn} = -\omega^2 \times r\cos(\alpha - \phi)$

$$= -(62.83)^2 \times 0.04 \times \cos(75° - 40°)$$
$$= -129.35 \text{ m/s}^2$$

At the top of the nose, acceleration, $a_{tn} = -\omega^2 \times r$

$$= -(62.83)^2 \times 0.04 = -157.9 \text{ m/s}^2$$

Spring force required to maintain contact between the follower and cam,

$$F = m \cdot a_{tn} = 5 \times 157.9 = 789.5 \text{ N}$$

Acceleration at beginning of the flank,

$$a_{sf} = \omega^2 \times OC\cos\phi$$
$$= (62.83)^2 \times 0.06217 \times \cos 0°$$
$$= 245 \text{ m/s}^2$$

Acceleration at the end of the flank, $a_{ef} = \omega^2 \times OC\cos\phi$

$$= (62.83)^2 \times 0.06217 \times \cos 40°$$
$$= 188 \text{ m/s}^2$$

Force required to stop the acceleration of 188 m/s^2 and decelerate at 129.35 m/s^2,

$$F = m\left(a_{ef} - a_{sn}\right) = 5(188 + 129.35) = 1586.75 \text{ N}$$

7.13 RADIUS OF CURVATURE AND UNDERCUTING

There is no restriction on the radius of curvature of the cam profile, with a knife-edge follower. The cam profile must be convex everywhere for a flat-face follower. In the case of a roller follower, the concave portion of the cam profile must have a radius of curvature greater than that of the roller to ensure proper contact along the cam profile.

To determine the pitch surface of a disc cam with radial roller follower, let the displacement R of the centre of the follower from the centre of the cam be given by (Fig.7.31)

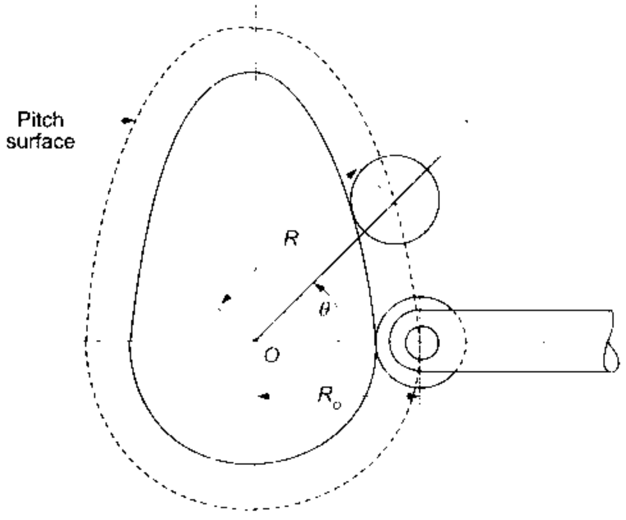


Fig.7.31 Disk cam with radial roller follower

$$R = R_o + f(\theta) \tag{7.46}$$

where R_o = minimum radius of the pitch surface of the cam

$f(\theta)$ = radial motion of the follower as a function of cam angle.

Once the value of R_o is known, the polar coordinates of the centers of the roller follower can be generated.

7.13.1 Kloomok and Muffley Method

Let ρ be the radius of curvature of the pitch surface

R_r be the radius of roller

These values are shown in Fig.7.32 together with the radius of curvature ρ_c of the cam surface. Here, ρ is held constant and R_r is increased so that ρ_c decreases. If this continued until R_r equals ρ, then ρ_c will be zero and the cam becomes pointed as shown in Fig.7.33(a). As R_r is further increased, the cam becomes undercut as shown in Fig.7.33(b), and the motion of the follower will not be as prescribed. Therefore, to prevent a point or an undercut from occuring on the cam profile, R_r must be les than ρ_{min}, where ρ_{min} is the minimum value of ρ over the particular segment of profile being considered. It is impossible to undercut a concave portion of a cam.

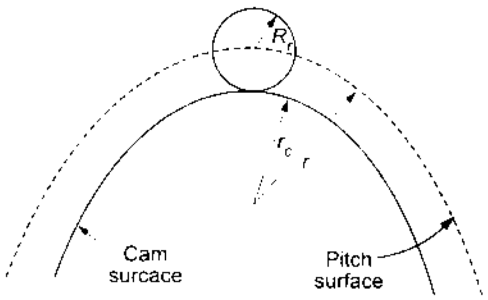
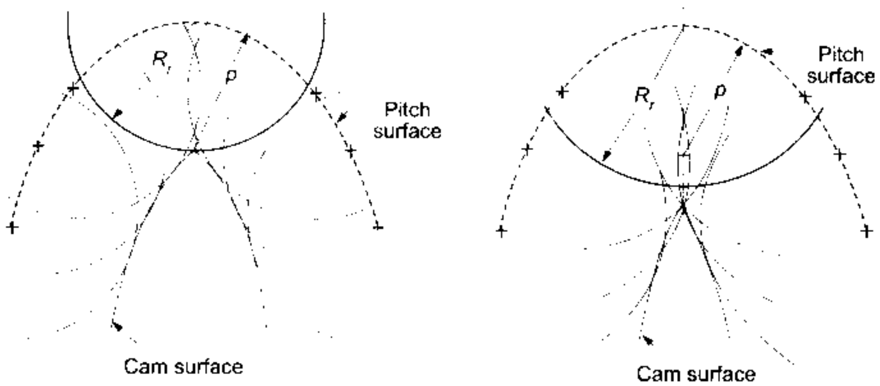


Fig.7.32 Cam and ritch surface



(a) Pointed cam (b) undercut cam

Fig.7.33 Undercutting in cams

The radius of curvature at a point on a curve can be expressed as:

$$\rho = \frac{\left[R_2 + \left(\frac{dR}{d\phi}\right)^2\right]^{\frac{3}{2}}}{\left[R^2 + 2\left(\frac{dR}{d\phi}\right)^2 - r\left(\frac{d^2R}{d\phi^2}\right)\right]}$$

where $R = f(\phi)$.

Here
$$f(\theta) = f(\phi)$$
$$\frac{dR}{d\theta} = f'(\theta)$$
$$\frac{d^2R}{d\theta^2} = f''(\theta)$$

Hence
$$\rho = \frac{\left[R^2 + \{f'(\theta)\}^2\right]^{\frac{3}{2}}}{\left[R^2 + 2\{f'(\theta)\}^2 - R\{f''(\theta)\}\right]} \tag{7.47}$$

For a convex cam surface, $R_r \leq r_{min}$

For concave cam surface, $\rho + R_r \geq R_{cutter}$

Pressure angle, $$\alpha = \tan^{-1}\left(\frac{1}{R} \cdot \frac{dR}{d\theta}\right) \quad (7.48)$$

7.13.2 Pressure Angle

The pressure angle is one of the most important parameters in cam design. By increasing the size of the cam, the pressure angle can be reduced.

Consider a cam with offset roller follower, as shown in Fig.7.34.

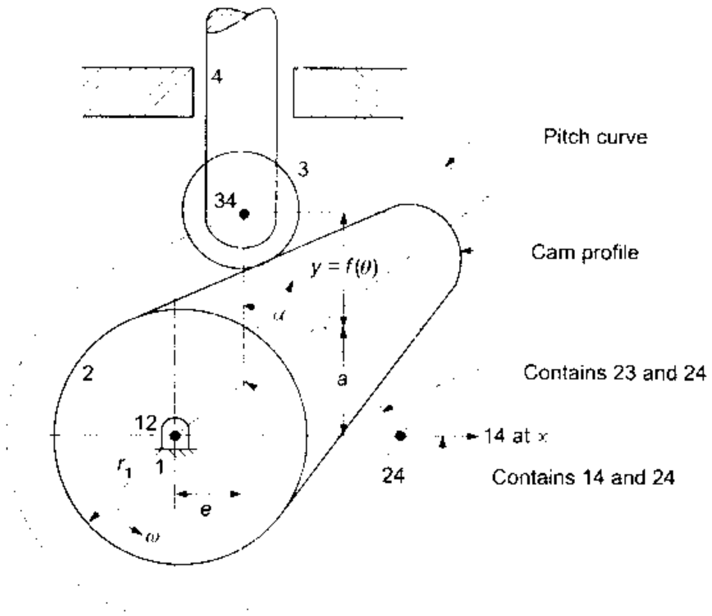


Fig.7.34 Cam with offset roller follower

Let α = pressure angle

r_1 = prime circle radius

e = offset

$y = f(\theta)$

θ = angle of cam rotation

ω = angular velocity of cam

The cam mechanism has four elements, namely, the fixed link 1, cam 2, roller 3, and follower rod 4. The instantaneous centres are as follows:

12 : at O

34 : at roller centre

14 : at infinity

23 : lies on the common normal at the point of contact of the roller and cam surface

24 : at the intersection of common normal at the point of contact and horizontal axis on which 14 lies.

As the movement of the follower rod is pure translation, all points on it have the same velocity. Thus, the velocity of follower during rise is:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = v_{24} = \omega(12 - 24) = \omega[e + (a + y)\tan\alpha] \tag{7.49}$$

where $a = \left(r_1^2 - e^2\right)^{0.5}$

$$\tan\alpha = \frac{\left[\frac{\mathrm{d}y}{\mathrm{d}t} - e\right]}{\left[f(\theta) + \left(r_1^2 - e^2\right)^{0.5}\right]} = \frac{\left[\frac{\mathrm{d}f}{\mathrm{d}\theta} - e\right]}{\left[f(\theta) + \left(r_1^2 - e^2\right)^{0.5}\right]} \tag{7.50}$$

During the return, we get

$$\tan\alpha = \frac{\left[\left|\frac{\mathrm{d}f}{\mathrm{d}\theta}\right| + e\right]}{\left[f(\theta) + \left(r_1^2 - e^2\right)^{0.5}\right]} \tag{7.51}$$

For maximum pressure angle, using rise, we have

$$\frac{\mathrm{d}(\tan\alpha)}{\mathrm{d}\theta} = \left[f(\theta) + \left(r_1^2 - e^2\right)^{0.5}\right]\left(\frac{\mathrm{d}^2 f}{\mathrm{d}\theta^2}\right) - \left(\frac{\mathrm{d}f}{\mathrm{d}\theta} - e\right)\frac{\mathrm{d}f}{\mathrm{d}\theta} = 0$$

$$\frac{\left(\frac{\mathrm{d}f}{\mathrm{d}\theta} - e\right)}{\left[f(\theta) + \left(r_1^2 - e^2\right)^{0.5}\right]} = \frac{\left(\frac{\mathrm{d}^2 f}{\mathrm{d}\theta^2}\right)}{\left(\frac{\mathrm{d}f}{\mathrm{d}\theta}\right)} = 0 \tag{7.52}$$

Solve (7.52) to get the value of θ_0. Then

$$\alpha_{\max} = \tan^{-1}\left[\left(\frac{\mathrm{d}^2 f}{\mathrm{d}\theta^2}\right) / \left(\frac{\mathrm{d}f}{\mathrm{d}\theta}\right)\right]_{\theta=\theta_0} \tag{7.53}$$

7.14 CAM SIZE

The cam size is defined by the following parameters:

1. Pressure angle
2. Radius of curvature of cam profile
3. Hub size

The following methods may be used to reduce the pressure angle:

1. Increase the diameter of the base circle.
2. Increase the angle of rotation of the cam, thereby lengthening the pitch curve for the specified follower displacement. The cam profile becomes flatter and the pressure angle becomes smaller.
3. Select the motion curve for a smaller pressure angle.
4. Change the offset of the follower.

Exercises

1 A disc cam is to give SHM to a knife edge follower during out stroke of 50 mm. The angle of ascent is 120°, dwell 60°, and angle of descent 90°. The minimum radius of the cam is 50 mm. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft and (b) when it is offset by 20 mm.

2 A disc cam with base circle 50 mm is operating a roller follower with SHM. The lift is 25 mm, angle of ascent 120°, dwell 60°, return 90°, and dwell during the remaining period. The cam rotates at 300 rpm. Find the maximum velocity and acceleration during ascent and descent. The roller radius is 10 mm. Draw the cam profile when (a) the line of reciprocation of follower passes through the cam axis and (b) when it is offset by 20 mm.

3 A cam with 30 mm as minimum diameter and 20 mm lift is rotating clockwise at a uniform speed of 1200 rpm and has to give the following motion to a roller follower 10 mm in diameter:

(a) Outward stroke during 120° with equal uniform acceleration and deceleration.

(b) Dwell for 60°.

(c) Return during 90°.

(d) Dwell during the remaining period.

Draw the cam profile if the cam axis coincides with the follower axis. Also calculate the maximum velocity and acceleration during ascent and return strokes.

4 Draw the cam profile in Exercise 3 when the follower is offset by 10 mm from the axis of the cam.

5 A flat-faced reciprocating follower has the following motion with uniform acceleration and retardation: ascent for 80°, dwell for 80°, and return for 120°. The base circle diameter of the cam is 60 mm and the stroke of the follower is 20 mm. The line of motion of the follower passes through the axis of the cam. Draw the cam profile.

6 A symmetrical tangent cam operating a roller follower has the following particulars:

Radius of base circle of cam	= 40 mm
Roller radius	= 20 mm
Angle of ascent	= 75°
Total lift	= 20 mm
Speed of cam shaft	= 300 rpm

Determine (a) the principal dimensions of the cam, (b) the equation of the displacement curve, when the follower is in contact with the straight flank and (c) the acceleration of the follower, when it is in contact with the straight flank where it merges into the circular nose.

7 A cam profile consists of two circular arcs of radii 30 mm and 15 mm, joined by straight lines, giving the follower a lift of 15 mm. The follower is a roller of 25 mm radius and its line of action is a straight line passing through the cam shaft axis. When the cam shaft has a uniform speed of 600 rpm, find the maximum velocity and acceleration of the follower while in contact with the straight flank of the cam.

8 The following data refers to a circular arc cam working with a flat-faced reciprocating follower: Minimum radius of cam = 30 mm, total angle of cam action = 120°, radius of circular arc = 80 mm and nose radius = 10 mm.

Find (a) the distance of the centre of the nose circle from the cam axis, (b) the angle through which the cam turns when the point of contact moves from the junction of minimum radius arc and circular arc to the junction of nose arc and circular arc and (c) velocity and acceleration of the follower when the cam has turned through an angle of 20°. The angular velocity of the cam is 10 rad/s.

9 The suction valve of a four stroke petrol engine is operated by a circular arc cam with a flat-faced follower. The lift of the follower is 10 mm; base circle diameter of the cam is 40 mm and the nose radius is 2.5 mm. The crank angle when suction valve opens is 4° after top dead centre. When the suction valve closes, the crank angle is 50° after bottom dead centre. If the cam shaft rotates at 600 rpm, determine (a) the maximum velocity of the valve and (b) the maximum acceleration and retardation of the valve.

10 The follower of a tangent cam is operated through a roller of 50 mm diameter and its line of stroke passes through the axis of the cam. The minimum radius of the cam is 40 mm and the nose radius 15 mm. The lift is 25 mm. If the speed of the cam shaft is 600 rpm, calculate the velocity and acceleration of the follower at the instant when the cam is (a) in full lift position and (b) 20° from full lift position.

11 (a) A reciprocating roller follower has cycloidal motion and its stroke of 30 mm is completed in 90° of cam rotation. The follower is offset against the direction of rotation by 6.25 mm and the radius of the roller is 12.5 mm. Determine the base circle radius which would limit the pressure angle to 30°. Also discuss the conditions that would lead to undercutting in the present case. (b) Discuss the advantages of trapezoidal time–acceleration curve when used for cam design.

12 The cam shown in Fig.7.35 rotates about O at an uniform speed of 500 rpm and operates a follower attached to the roller with centre Q. The path of Q is a straight line passing through O. Draw the time-lift diagram of the roller centre on a base of 10 mm equaling 0.005 s and to a vertical scale of four times full size for a movement of 180° from the position shown. Find also the cam angle for which the velocity of the follower is a maximum and determine the magnitude of the velocity.

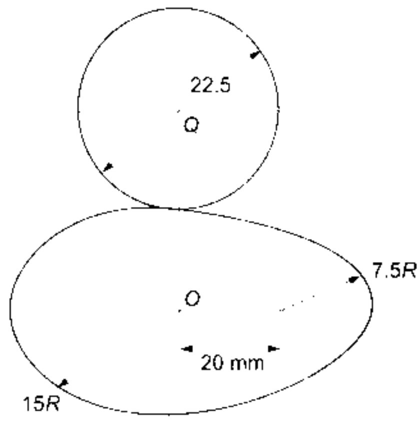


Fig.7.35

13 What is meant by pressure angle of a cam? Discuss its significance. For a knife-edge parabolic cam follower system, show that $\tan \phi_m = \frac{1}{\theta_m}$ when the follower is under constant acceleration from rest and where

ϕ_m is the maximum pressure angle and

θ_m is the angle through which the cam turns

while the follower rises from rest to where the pressure angle is maximum. State the assumptions made.

14 Distinguish between the functions of a cam and an eccentric. What are the considerations that govern the choice of profile of a cam? To what extent can these considerations be satisfied?

15 A cam having a lift of 10 mm operates the suction valve of a four stroke SI engine. The least radius of the cam is 20 mm and nose radius is 2.5 mm. The crank angle for the engine when suction valve opens is 4° after *TDC* and it is 50° after *BDC* when the suction valve closes. The crank shaft speed is 2000 rpm. The cam is of circular type with circular nose and flanks. It is integral with cam shaft and operates a flat-faced follower. Estimate (a) the maximum velocity of the valve, (b) the maximum acceleration and retardation of the valve, and (c) the minimum force to be exerted by the spring to overcome inertia of the valve parts which weigh 2 N.

16 (a) How is the displacement diagram drawn for a cam with parabolic motion of its follower if the motion is with uniform acceleration? Explain the pressure angle for a cam follower system. How does the pressure angle influence the jamming of the translating follower?

(b) What is pressure angle in cam action? How is it important in cam design?

17 Draw the profile of a cam to raise a valve with harmonic motion through 50 mm in 1/3rd of a revolution keep it fully raised through 1/12th of a revolution and to lower it with harmonic motion in 1/6th of a revolution. The valve remains closed during the rest of the revolution. The diameter of the roller is 20 mm and the minimum radius of the cam is to be 25 mm. The diameter of the cam shaft is 25 mm. The axis of the valve rod passes through the axis of the cam shaft. Assume the camshaft to rotate with uniform velocity.